Cosmology from matrix model formulation of superstring theory

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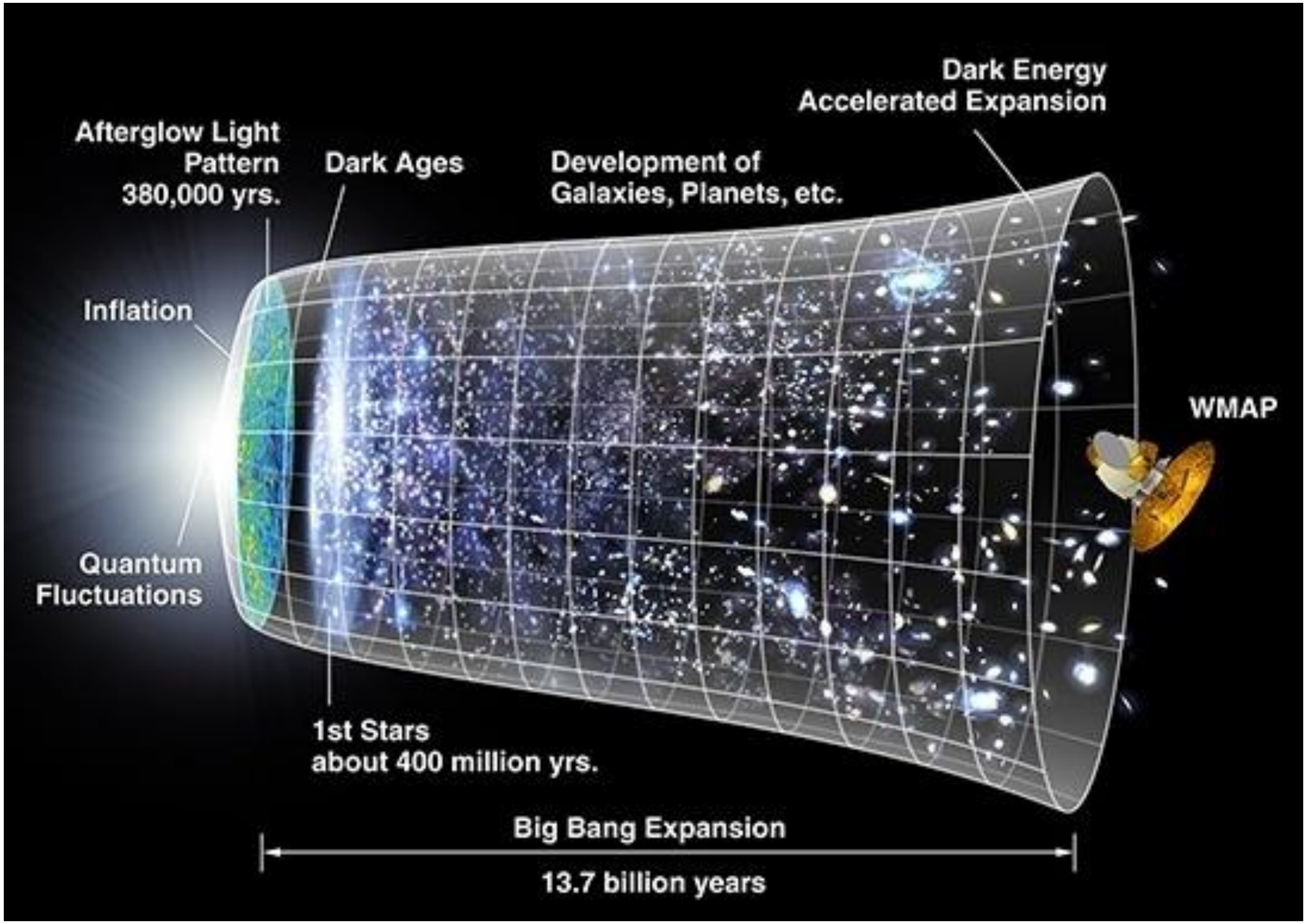
Invited talk at Tsukuba Global Science Week 2018
Tsukuba International Congress Center, Tsukuba, Japan

September 21, 2018

Big Bang cosmology



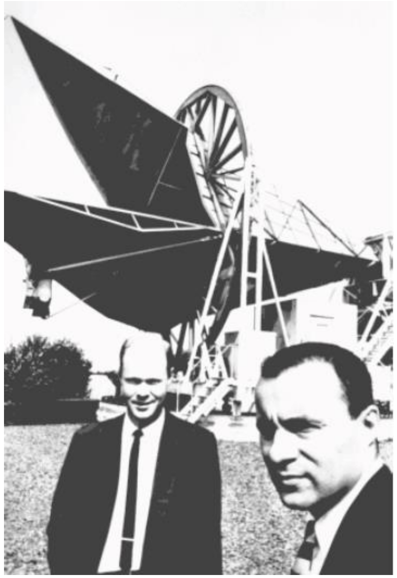
Our universe is expanding since it was born as an invisibly tiny point 13.7 billion years ago.

3 evidences for standard Big Bang cosmology

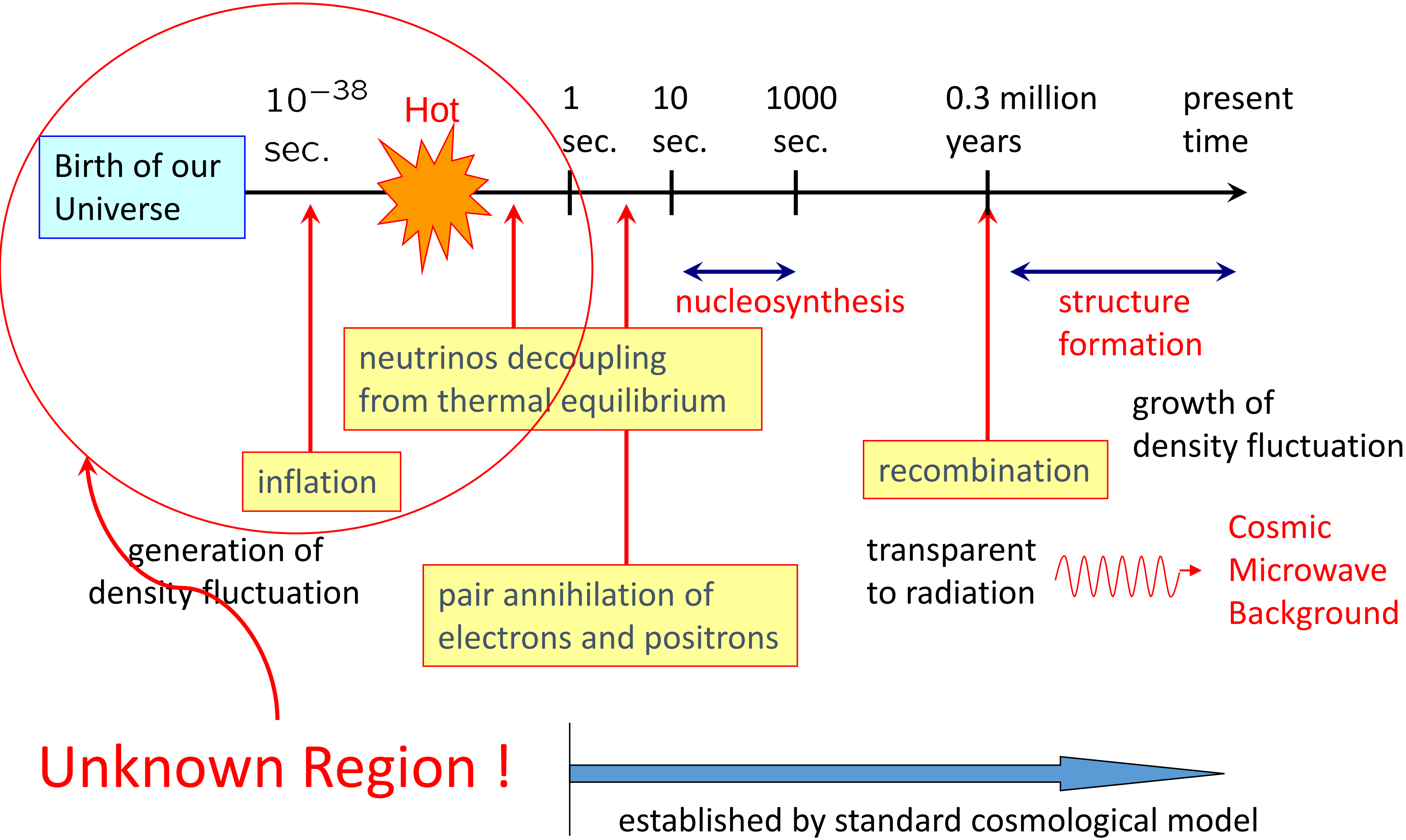
- Discovery of cosmic expansion (Hubble 1929)
- Theory of nucleosynthesis (Alpher-Bethe-Gamov 1948)
- Discovery of Cosmic Microwave Background (Penzias-Wilson 1965)


Hubble


Gamov


Penzias, Wilson

History of our Universe



Birth of our Universe

10^{-38} sec.

Hot

1 sec.

10 sec.

1000 sec.

0.3 million years

present time

nucleosynthesis

structure formation

neutrinos decoupling from thermal equilibrium

inflation

growth of density fluctuation

recombination

generation of density fluctuation

pair annihilation of electrons and positrons

transparent to radiation

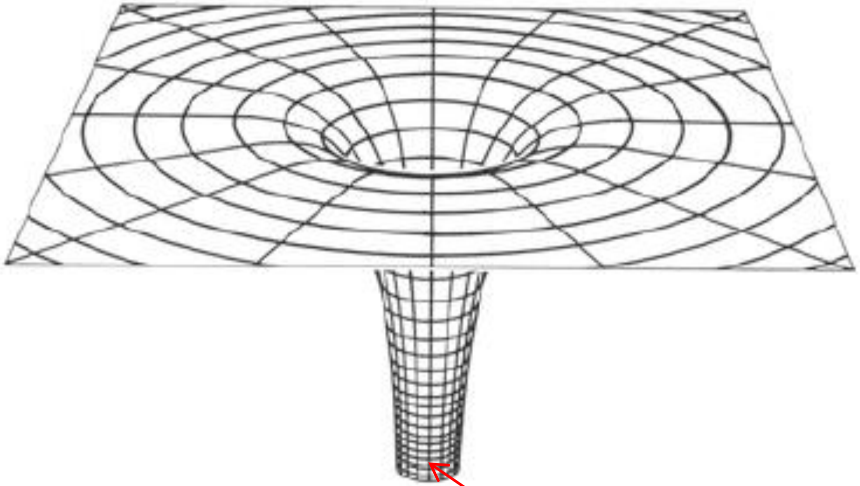
Cosmic Microwave Background

Unknown Region !

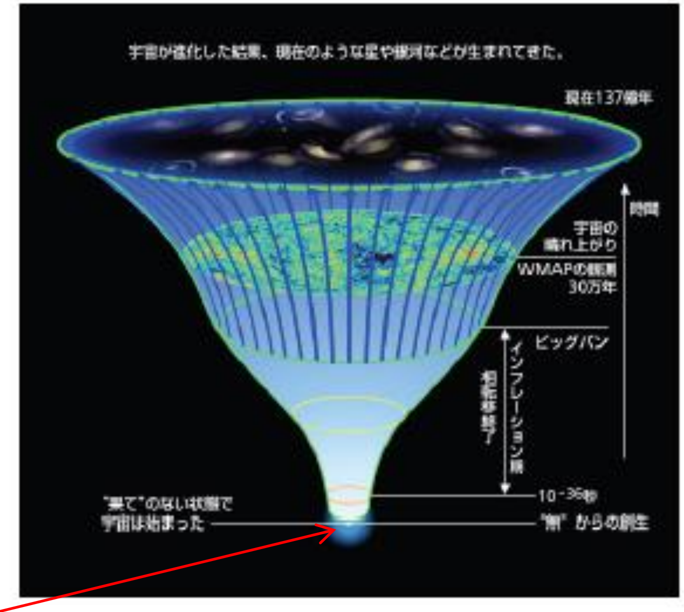
established by standard cosmological model

Singularities (space-time curvature diverges)

Black hole



Big bang



singularity (curvature diverges)

General Relativity becomes invalid!

（Quantum effects become non-negligible.）

We need to go beyond Einstein !

Quantum Cosmology

- “Creation of Universes from nothing” Vilenkin (’82, ’84)

  tunneling effects discussed with imaginary time

- “Wave function of the Universe” Hartle-Hawking (’83)

  “no boundary” proposal in path-integral formulation

The problem of UV divergences in quantum gravity
was ignored by restriction to a uniform isotropic universe.

a consistent theory of quantum gravity : superstring theory

We investigate the beginning of our Universe
using its matrix formulation.

Plan of the talk

0. Introduction
1. Superstring theory as a consistent theory of quantum gravity
2. Matrix Model for superstring theory
3. The beginning of our Universe
4. Expanding behaviors in simplified models
5. Summary

1. Superstring theory as a consistent theory of quantum gravity

Superstring theory

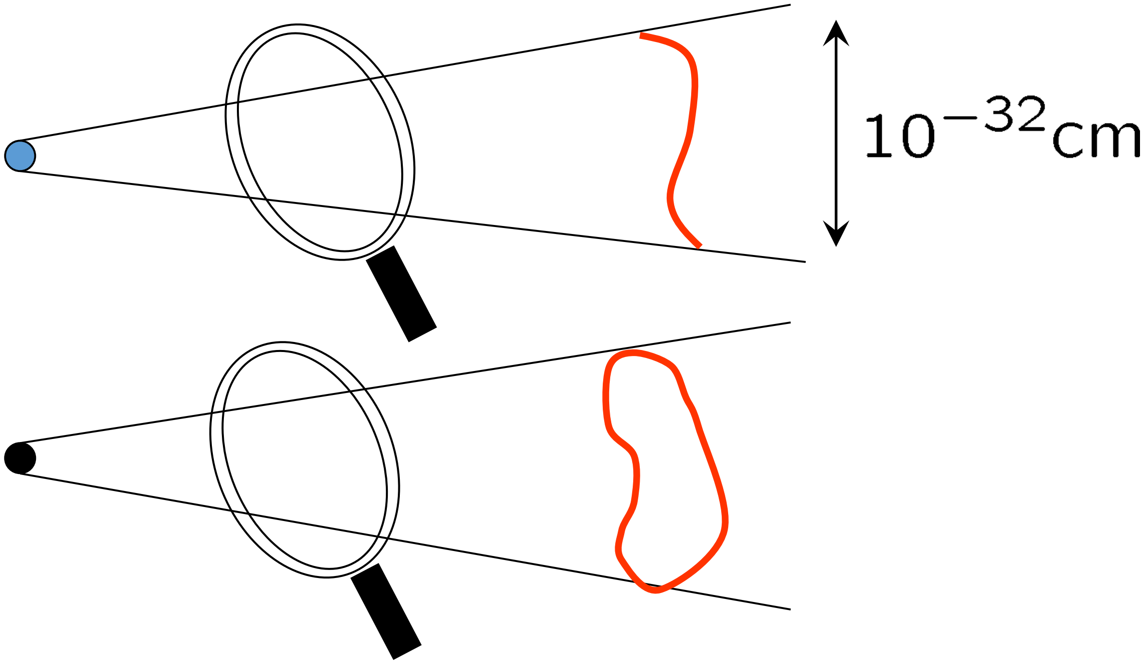
1974 Sherk-Schwarz, Yoneya
1984 Green-Schwarz

Quantum gravity is non-renormalizable due to severe UV divergences.

This problem can be solved by describing particles by extended objects like strings !

photon
gluon
$W^{\pm},\ Z$ etc.

graviton,
in particular



Various vibration modes correspond to various particles.

Unified description of 4 forces including gravity

The goals of superstring theory

particle physics

- space-time dimensionality puzzle

  critical dimension is (9+1), but we live in (3+1)d

- particle contents

  gauge group : $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$
  matter contents : 3 generations $(q \text{ and } \ell)$ + Higgs

- coupling constants in the Standard Model

cosmology

- the birth of our Universe and "inflation"
- the fate of our Universe

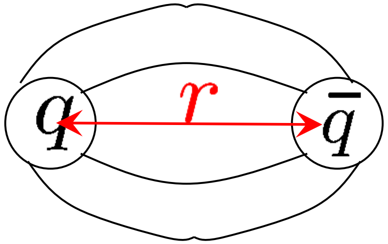
  (dark energy, cosmological constant problem)

- the interior structure of a black hole

A big obstacle:

non-perturbative definition is not yet established !

c.f.) quark confinement in QCD



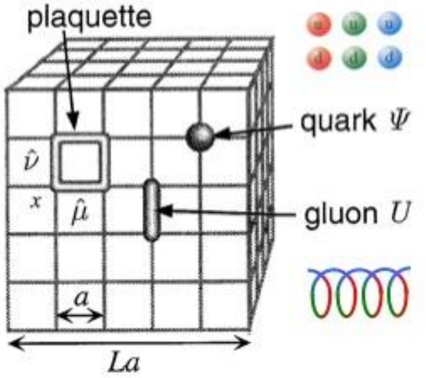
$V(r) \propto r$

$q\bar{q}$ potential

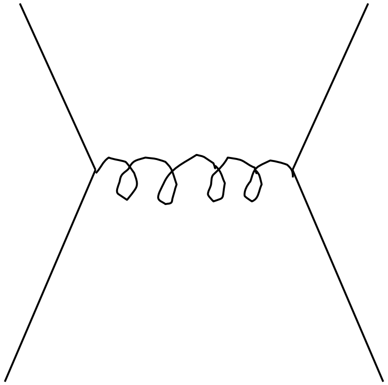
$V(r) \propto -\frac{1}{r}$

non-perturbative calculations

perturbative calculations



lattice gauge theory (Wilson, 1974)



Compactification

- Superstring theory is naturally defined in (9+1)dim.

  unitarity + Lorentz invariance

- (3+1)-dimensional space-time is expected to appear due to some non-perturbative dynamics.

  not well understood yet.

- Search for perturbative vacua with compactified 6d.

  good : One can obtain Standard Model-like models.
  bad : Too many vacua. ("Landscape")

Understanding the non-perturbative dynamics of compactification is crucial to understand our real world !

2. Matrix model for superstring theory

Matrix model for superstring theory in 10d

- IKKT matrix model　　Ishibashi-Kawai-Kitazawa-Tsuchiya ('97)

$$
\begin{aligned}
S_{\mathrm{b}} &= -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu]) \\
S_{\mathrm{f}} &= -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\,\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])
\end{aligned}
$$

proposed as a non-perturbative formulation of superstring theory

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

raised and lowered by the metric

$$\eta = \mathrm{diag}(-1, 1, \cdots, 1)$$

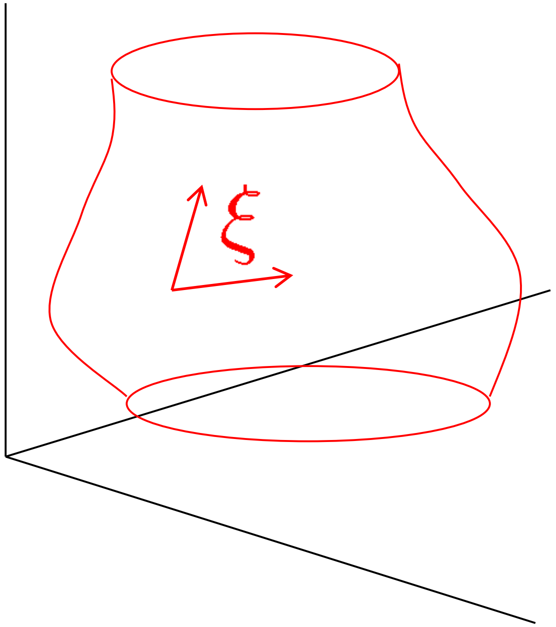
The action has manifest SO(9,1) symmetry and maximal SUSY.

eigenvalues of $A_\mu \Longrightarrow$ dynamical space-time.

Connection to the perturbative formulation

- worldsheet theory (1st quantization)

$X^\mu(\xi),\ \Psi(\xi)$



$$S = \int d^2\xi\sqrt{g}\left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\}\right)$$

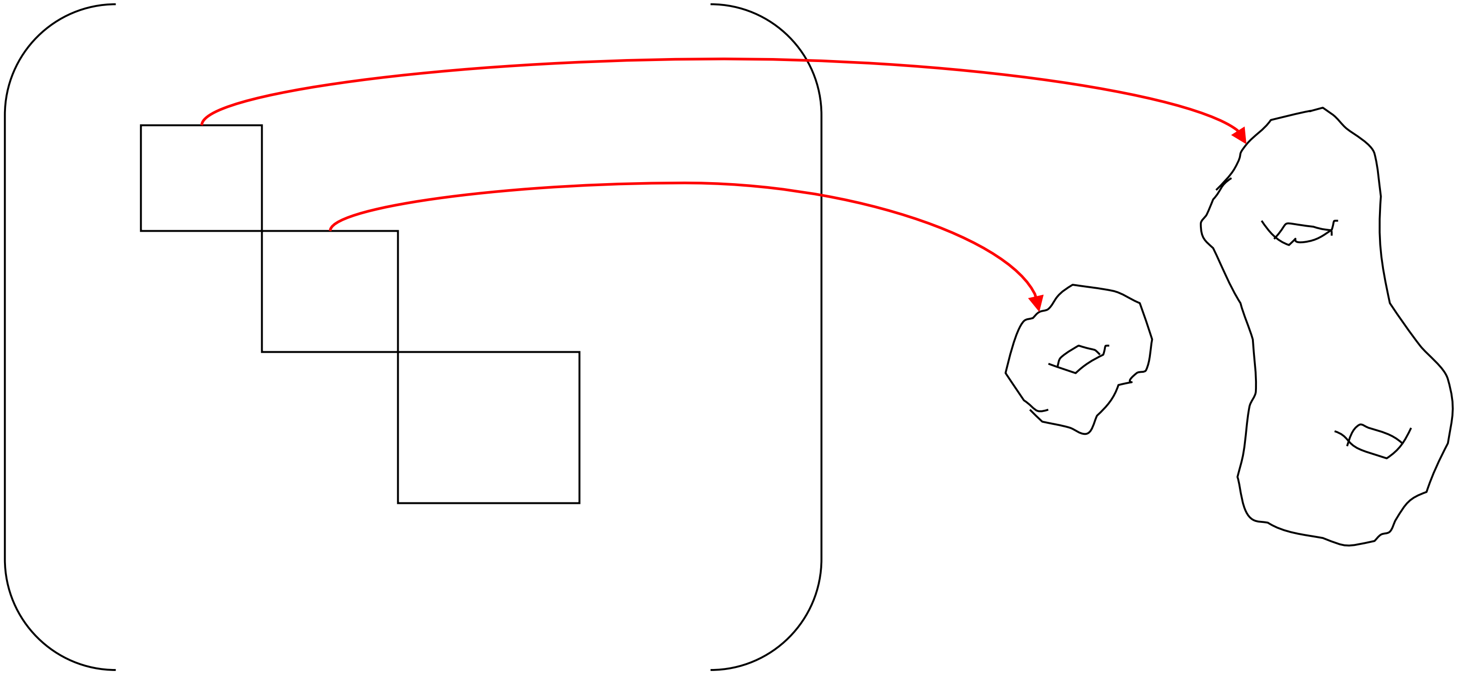
$$\{X, Y\} \equiv \frac{1}{\sqrt{g}}\epsilon^{ab}\frac{\partial X}{\partial\xi^a}\frac{\partial Y}{\partial\xi^b}$$

Poisson bracket (regarding ξ_1 and ξ_2 as p and q in Hamilton dynamics)

quantization $\Longrightarrow$ IKKT action $\quad (\hbar \sim \frac{1}{N})$

$$\{X^\mu(\xi), X^\nu(\xi)\} \mapsto -i[A^\mu, A^\nu]$$

Natural realization of 2nd quantization



Each of these blocks ⟺ disconnected worldsheet

many-body states of strings naturally included (2nd quantization !)

c.f.) Schwinger-Dyson equations in the IKKT model

→ Hamiltonian of String Field Theory Fukuma-Kawai-Kitazawa-Tsuchiya ('98)

3. The beginning of our Universe

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601

The definition of the partition function

connection to the worldsheet theory

$$Z = \int dA\, d\Psi\, e^{iS} = \int dA\, e^{iS_{\mathrm{b}}}\, \underbrace{\mathrm{Pf}\,\mathcal{M}(A)}_{\text{polynomial in } A}$$

pure phase factor

We have to regularize the model by introducing IR cutoffs.

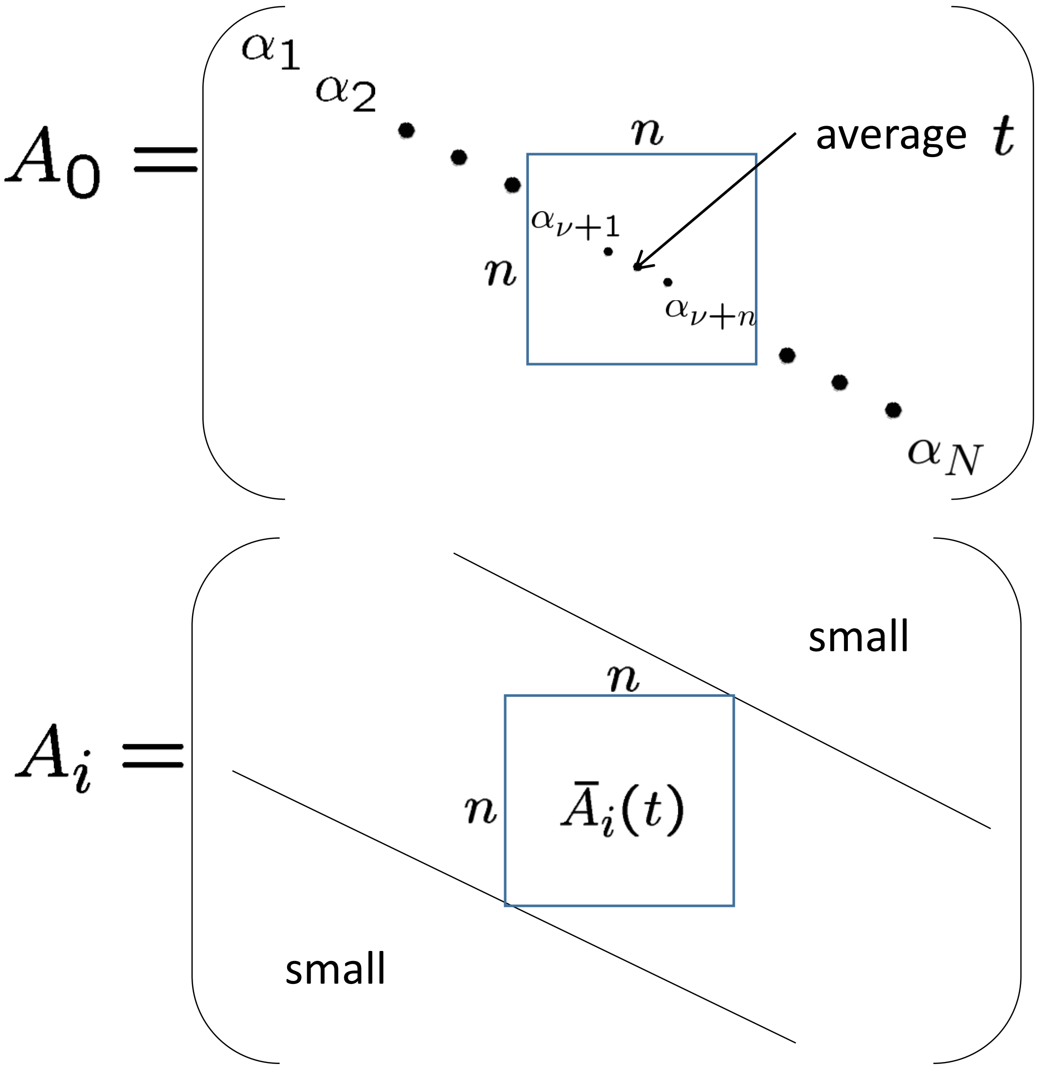
$$\begin{aligned} \frac{1}{N}\mathrm{tr}\,(A_0)^2 &\leq \kappa L^2 \\ \frac{1}{N}\mathrm{tr}\,(A_i)^2 &\leq L^2 \end{aligned}$$

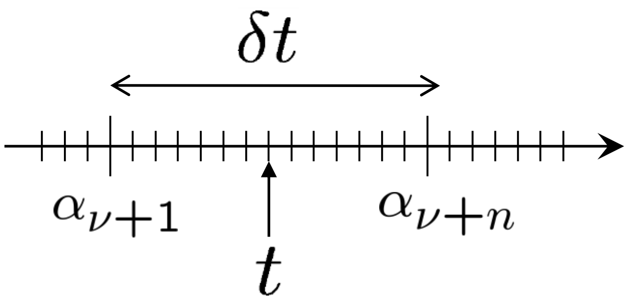
Note also : $S_{\mathrm{b}} \propto \mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu}) = \underbrace{-2\,\mathrm{tr}\,(F_{0i})^2 + \mathrm{tr}\,(F_{ij})^2}_{\text{opposite signs !}}$

$F_{\mu\nu} = i[A_\mu, A_\nu]$: Hermitian

How to extract the real-time evolution from the matrix configurations

diagonalize A_0 using SU(N) sym : $A_\mu \rightarrow U A_\mu U^\dagger$





$$\nu = 0, 1, \cdots, N - n$$

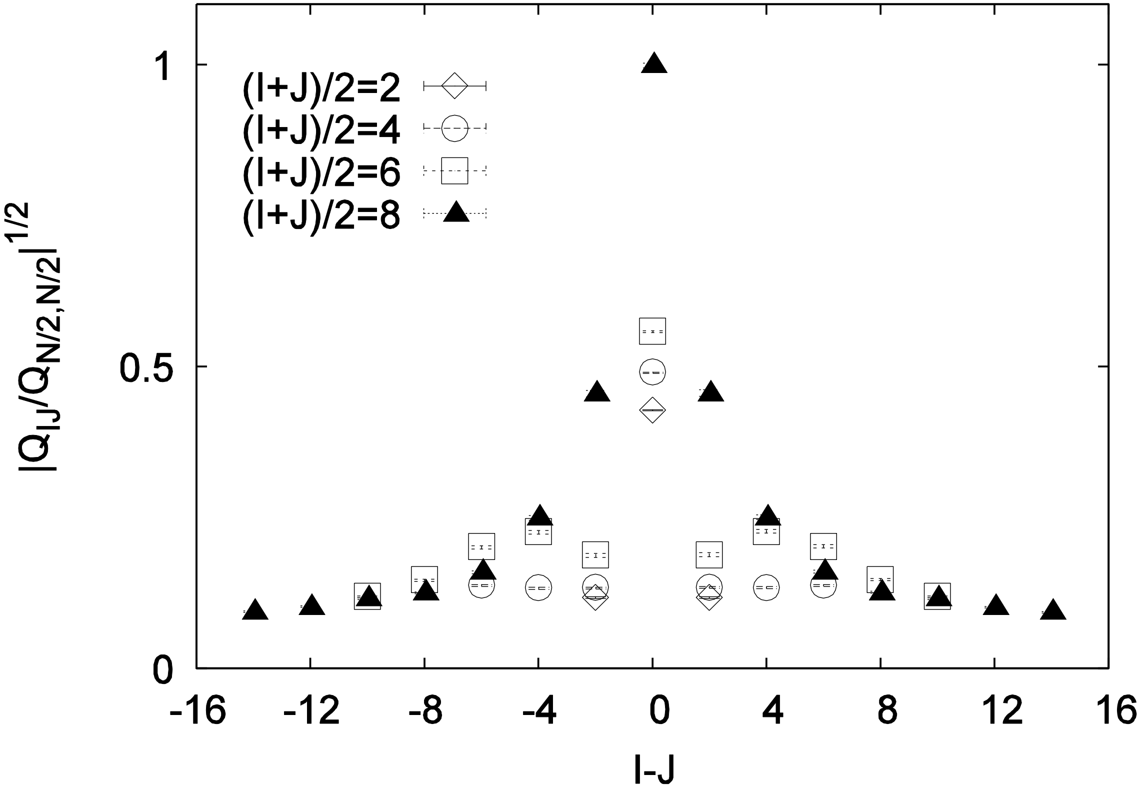
$$t = \frac{1}{n} \sum_{a=1}^{n} \alpha_{\nu+a}$$

We observe
band-diagonal structure.

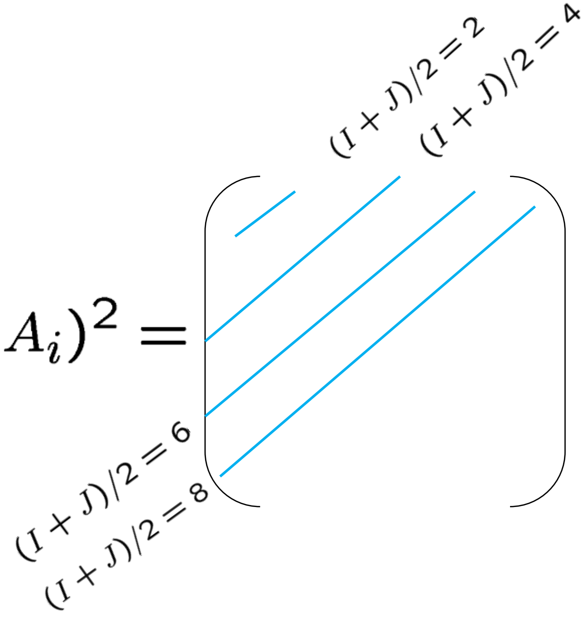
$\bar{A}_i(t)$ represents the state at fixed time t.

Determination of the block size

$N = 16$



$$(A_i)^2 =$$



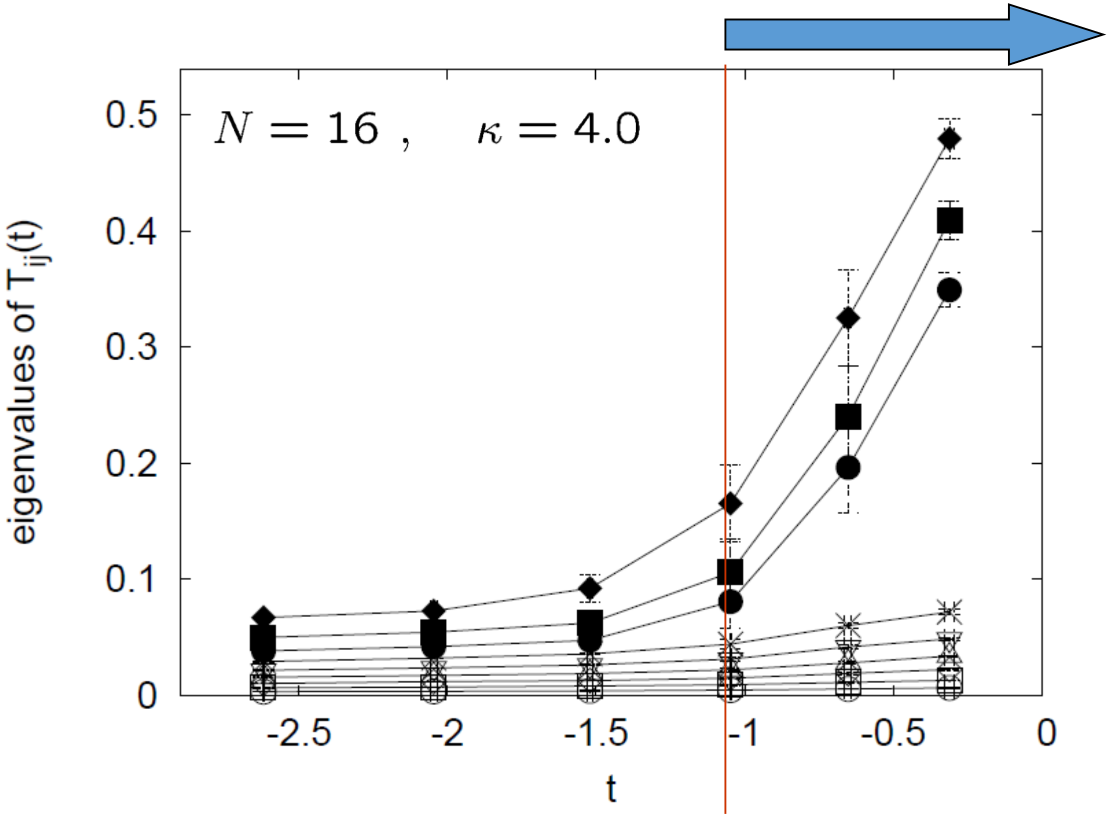
We take $n = 4$.

Spontaneous breaking of SO(9)

Eigenvalues of $T_{ij}(t) = \frac{1}{n}\text{tr}\left\{\bar{A}_i(t)\bar{A}_j(t)\right\}$

represent the extent of space-time in each direction.

$\text{SO}(9) \overset{\text{SSB}}{\rightarrow} \text{SO}(3)$



"critical time"

Mechanism of SSB (a simple example)

$$\begin{aligned}&\text{maximize } \frac{1}{N}\mathrm{tr}\,(F_{ij})^2\\ &\text{with fixed } \frac{1}{N}\mathrm{tr}\,(A_i)^2 = 1\end{aligned}$$

$$\text{maximize } G = \mathrm{tr}\,(F_{ij})^2 - \lambda\,\mathrm{tr}\,(A_i)^2$$

Lagrange multiplier

$$\Downarrow$$

$$2\,[A_j,[A_j,A_i]] - \lambda A_i = 0$$

solution :

$$\begin{aligned}A_i &= \chi L_i &&\text{for } i \le d\\ A_i &= 0 &&\text{for } d < i \le 9\end{aligned}$$

representation matrices of a compact semi-simple Lie algebra with d generators

$$\Rightarrow \quad L_i = \begin{pmatrix} \frac{1}{2}\sigma^i & \\ & 0 \end{pmatrix} \quad \Rightarrow \quad \boxed{d=3\ !}$$

Examples of spontaneous symmetry breaking in theoretical physics

- superconductor (Bardeen-Cooper-Schrieffer theory)
  SSB of U(1) sym. due to formation of Cooper pairs

- the origin of constituent quark mass in QCD (Nambu)
  SSB of chiral sym. due to formation of chiral condensate
  pions as Nambu-Goldstone bosons

Likewise, we may say :

The origin of our Universe :
SSB of SO(9,1) in the IKKT matrix model

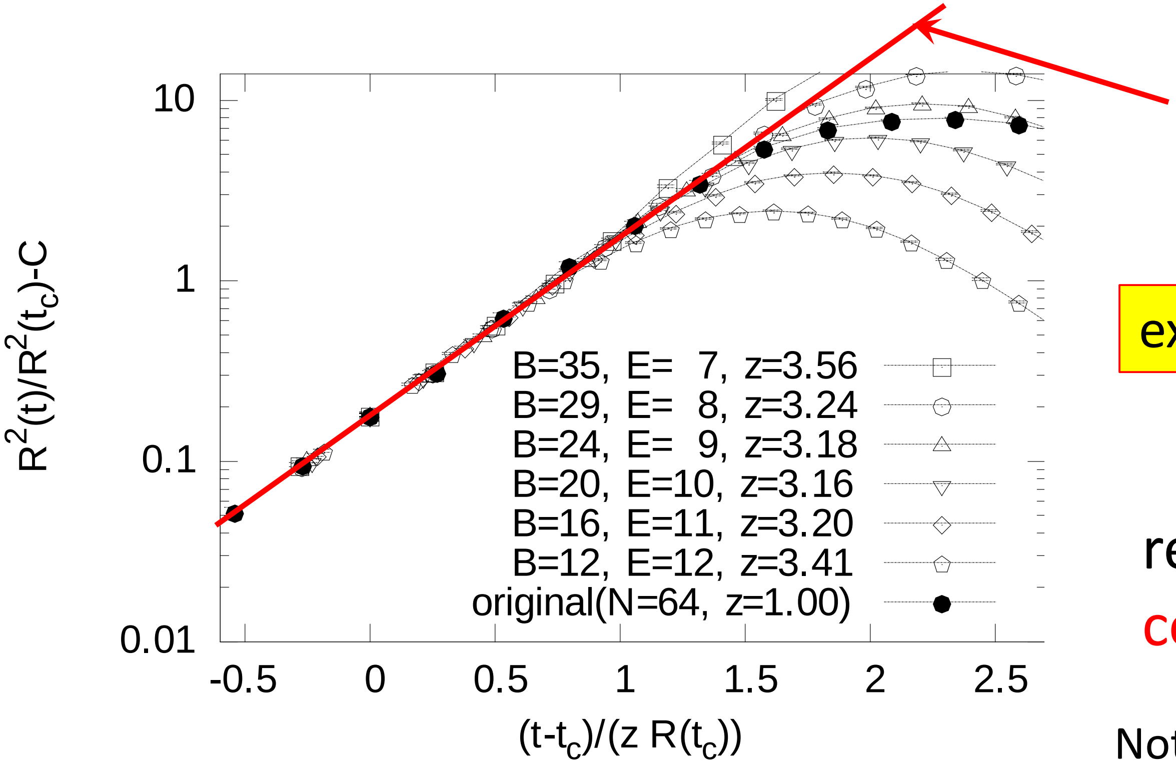
4. Expanding behaviors in simplified models

Ito, Kim, Koizuka, J.N., Tsuchiya, PTEP 2014 (2014) no.8, 083B01
Ito, J.N., Tsuchiya JHEP 1511 (2015) 070

Exponential expansion at early times

results of a simplified model for early time behaviors

Ito, Kim, Koizuka, J.N., Tsuchiya
PTEP 2014 (2014) no.8, 083B01



$R(t) \propto e^{\alpha t}$

exponential expansion

⇩

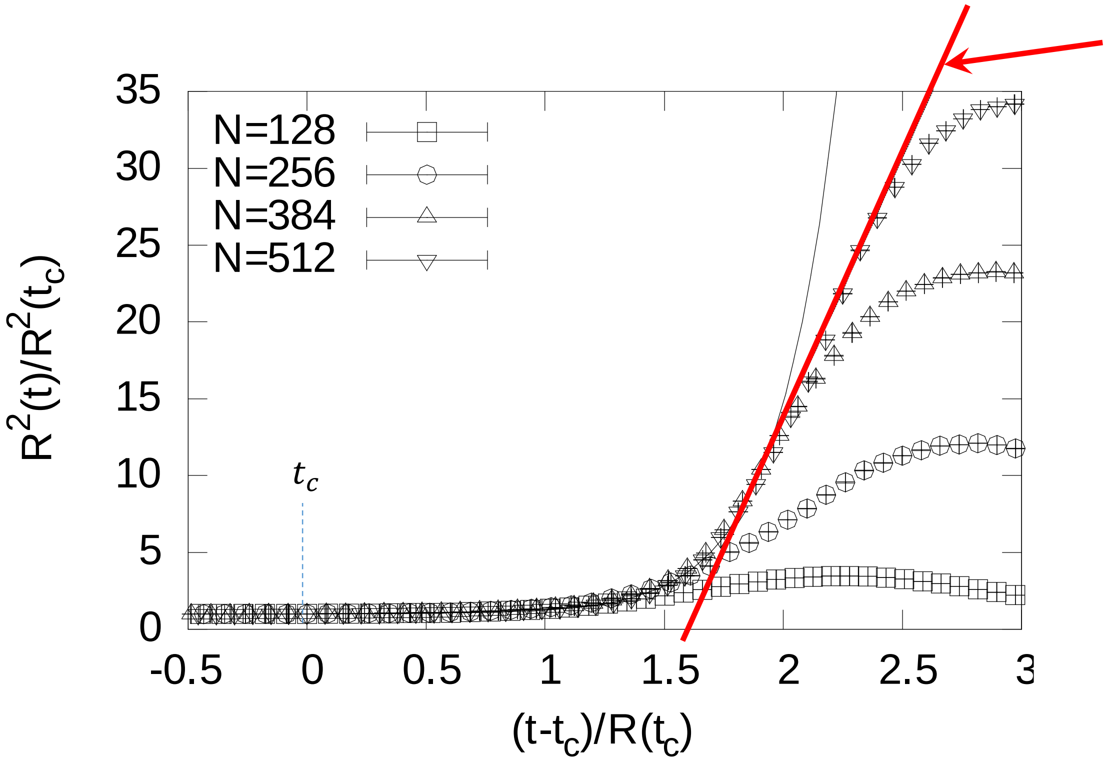
reminiscent of the
cosmic inflation

Note: No inflaton, no potential
introduced by hand.

Power law expansion at late times

results of a simplified model for late time behaviors

Ito, J.N., Tsuchiya JHEP 1511 (2015) 070

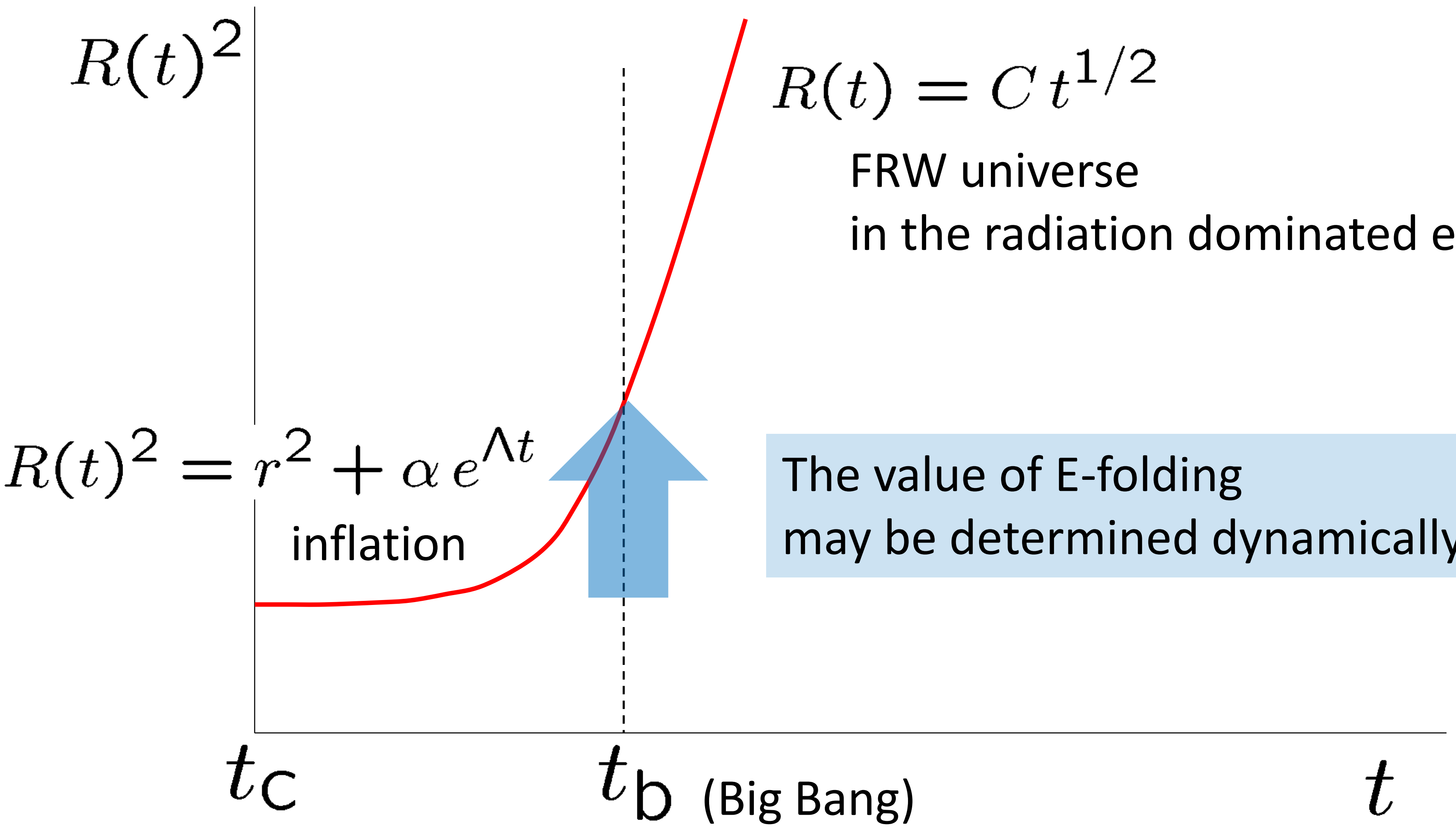


$$R^2 \sim t \quad \Rightarrow \quad R(t) \propto t^{1/2}$$

power law expansion

reminiscent of the Friedmann-Robertson-Walker Universe in the radiation dominated era

The expected scenario for the full IKKT matrix model



The value of E-folding
may be determined dynamically !

5. Summary and future prospects

Summary

- Superstring theory
  - strings naturally solve the problem of severe UV divergences in quantum gravity
  - unified theory of all particles (both forces and matters)
  - however, too many vacua (“landscape”)
    due to variety of compactifications from 10d to 4d
  - fully non-perturbative formulation is crucial

- Matrix model formulation
  - analogous to lattice gauge theory for QCD
  - IKKT model : non-perturbative formulation of superstrings
  - Monte Carlo simulation revealed
    SSB of SO(9) down to SO(3) at some critical time.
  - expanding behavior in simplified models
    exponential/power-law

Future prospects

- Do Standard Model particles (gauge bosons, quarks, leptons, Higgs) appear at later times ?

Monte Carlo simulation &
studies of classical solutions (+ quantum corrections)

Classical eq. of motion becomes valid at late times
due to the expansion of space.

We hope the investigations of the IKKT matrix model
will provide a new perspective on

particle physics beyond the standard model (incl. dark matter)
cosmological models for inflation, dark energy, etc..