Charm and beauty in the deconfined plasma from quenched lattice QCD

PRD 92 (2015) no.11, 116003, arXiv:1508.04543 [hep-lat]
JHEP 11 (2017) 206, arXiv:1709.07612 [hep-lat]
arXiv:2108.13693 [hep-lat]

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宇宙史研究センター2021年度
第2回構成員会議・成果報告&交流会
2021年11月26日

Outline

- Introduction
  - Heavy flavor, spectral function
- Lattice studies
  - Color electric correlators and spectral functions
  - Quarkonium correlators and spectral functions in quenched QCD (pseudo-scalar and vector channels)
- Summary

Heavy flavor

- Heavy flavor = charm & bottom

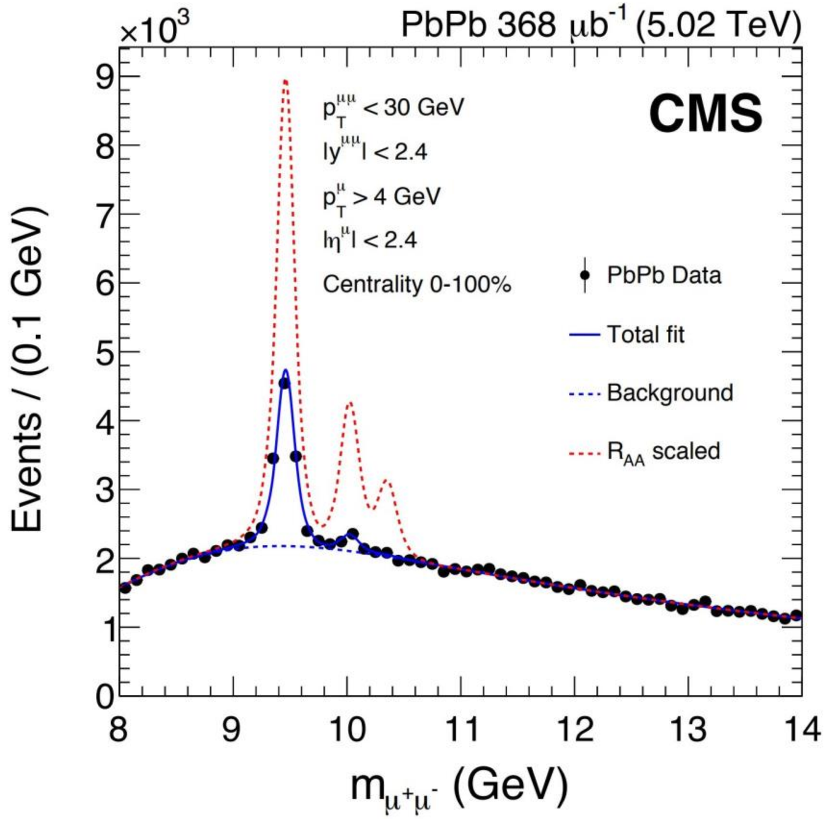
η_c, J/Ψ, χ_c, η_b, Υ, χ_b …

D, B, …

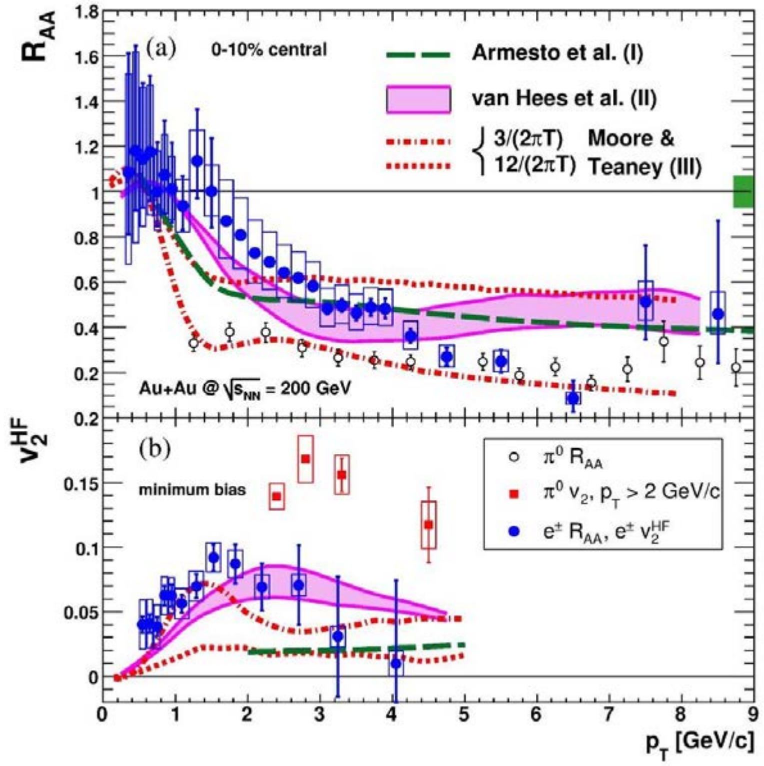
Q = c, b
q = u, d, s

- **Quarkonia ($Q\bar{Q}$)** and **open heavy flavors ($Q\bar{q}$, $q\bar{Q}$)** are important probes to investigate **Quark-Gluon Plasma (QGP)** formed in heavy ion collisions.
  – Produced in the early stage of the collisions: experiencing entire evolution of QGP
  – A signal of QGP formation: color Debye screening in QGP
    → suppression of quarkonium production   T. Matsui and H. Satz, PLB 178 (1986) 416
  – Sequential suppression: different binding energy for different bound state
    → different melting temperature

What should we understand?



CMS Collaboration@QM2018



PHENIX Collaboration, PRL 98 (2007) 172301

Understanding suppression patterns
→ **dissociation temperatures**

Inputs for transport models
→ **heavy quark diffusion coefficients**

In-medium properties of open/hidden heavy flavors ← All encoded in **spectral functions**

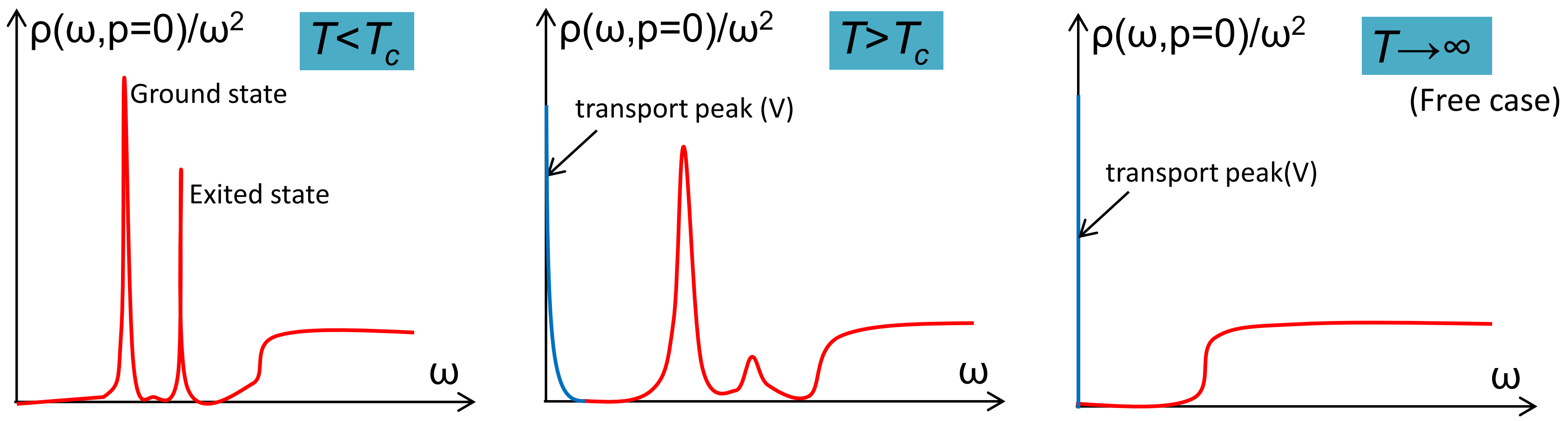
Spectral function

Euclidian (imaginary time) mesonic correlation function

$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0})\rangle \qquad J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x})\Gamma_H \psi(\tau,\vec{x})$$

$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega,\vec{p}) K(\omega,\tau) \qquad K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)}$$

Spectral function



Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho^V_{ii}(\omega,\mathbf{0})}{\omega}$$

ρ^V_{ii} : Vector spectral function
χ_{00} : Quark number susceptibility

Difficulties on the lattice

- **Heavy quark mass m_Q is too heavy!**
  - Fine and large lattices are needed to control lattice artifacts.

- **Obtaining spectral functions: an ill-posted inverse problem!**
  - # of correlator data points << # of frequency bins of spectral functions
  - A naive χ^2-fiting gives infinite number of possible spectra within statistical uncertainties.

Overcoming difficulties

- **Heavy quark mass m_Q is too heavy!**
  - $m_Q >> \Lambda_{QCD} \rightarrow$ separation of scales (especially for bottom)
  - Effective field theories: **Non-relativistic QCD (NRQCD), potential NRQCD (pNRQCD)**, …

Review: N. Brambilla *et al.*, Rev. Mod. Phys. 77 (2005) 1423

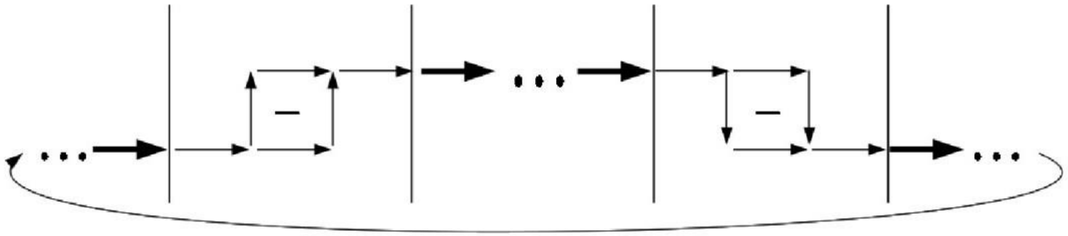
- **Obtaining spectral functions: an ill-posted inverse problem!**
  - Adding prior information
  - Bayesian inference: **Maximum Entropy Method (MEM), Bayesian Reconstruction (BR) Method, …**
  - **Phenomenologically motivated (perturbative) modeling of spectral functions**
  - Other approached without spectral functions

Color electric correlator

- Heavy quark effective theory → color electric correlator

S. Caron-Huot, M. Laine and G.D. Moore, JHEP 04 (2009) 053

$$G_E(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\beta;\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}\left[U(\beta;0)\right]\right\rangle}, \quad \beta \equiv \frac{1}{T}$$



$$G_E(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_E(\omega) \frac{\cosh[\omega(\frac{\beta}{2}-\tau)]}{\sinh[\frac{\omega\beta}{2}]}.$$

- Heavy quark momentum diffusion coefficient

$$\kappa \equiv \lim_{\omega\to 0} \frac{2T\rho_E(\omega)}{\omega}$$

Perturbative model spectral function

- Modeling the spectral function
  - IR part $(\omega \ll T)$

A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

$$\kappa \equiv \lim_{\omega \to 0} \frac{2T\rho_E(\omega)}{\omega} \quad \rightarrow \quad \phi_{IR}(\omega) \equiv \frac{\kappa\omega}{2T}$$

  - UV part $(\omega \gg T)$

: computed from perturbation theory, two different ansatzes

$$\phi_{UV}^{(a)}(\omega) \qquad \phi_{UV}^{(b)}(\omega)$$

  - Interpolations

: two different interpolations, three different models

$$\rho_E^{(1\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{max}} c_n e_n^{(\mu)}(y)\right]\left[\phi_{IR}(\omega) + \phi_{UV}^{(i)}(\omega)\right] \qquad \rho_E^{(2\mu i)}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{max}} c_n e_n^{(\mu)}(y)\right]\sqrt{\left[\phi_{IR}(\omega)\right]^2 + \left[\phi_{UV}^{(i)}(\omega)\right]^2}$$

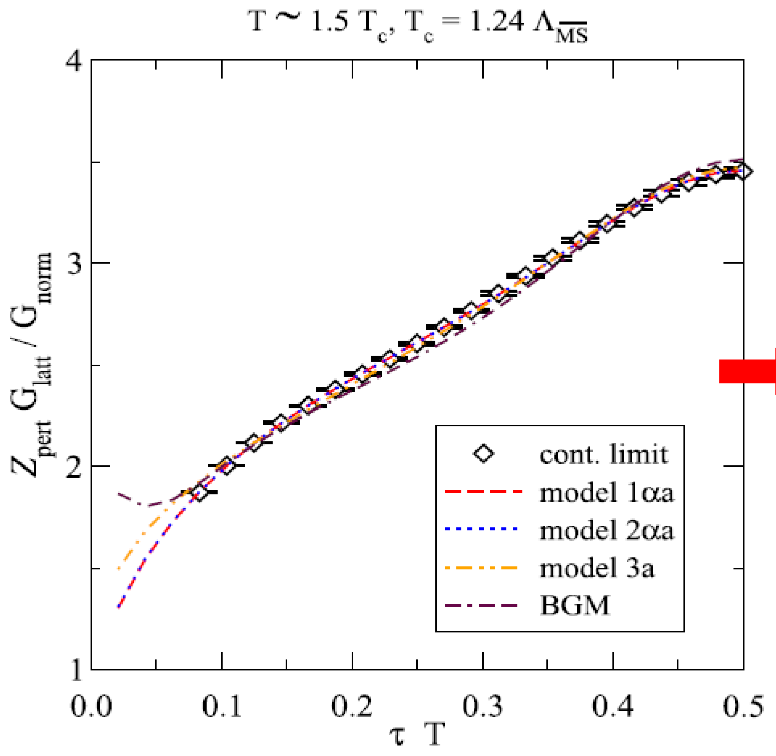
$$\rho_E^{(3i)}(\omega) \equiv \max\left[\phi_{IR}(\omega), c\,\phi_{UV}^{(i)}(\omega)\right]$$

Heavy quark momentum diffusion coefficient

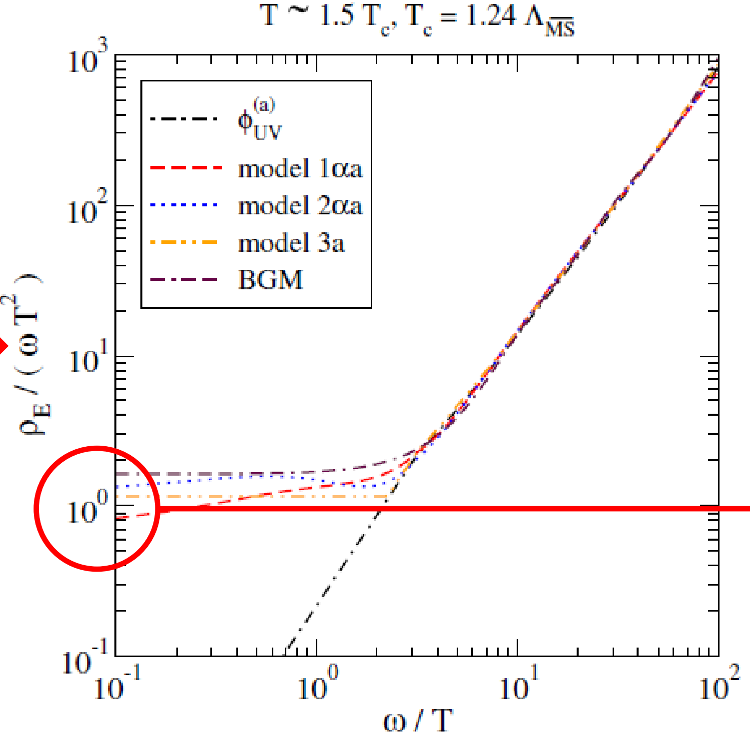
A. Francis, O. Kaczmarek, M. Laine, T. Neuhaus and HO, PRD 92 (2015) no.11, 116003

$1.5T_c$

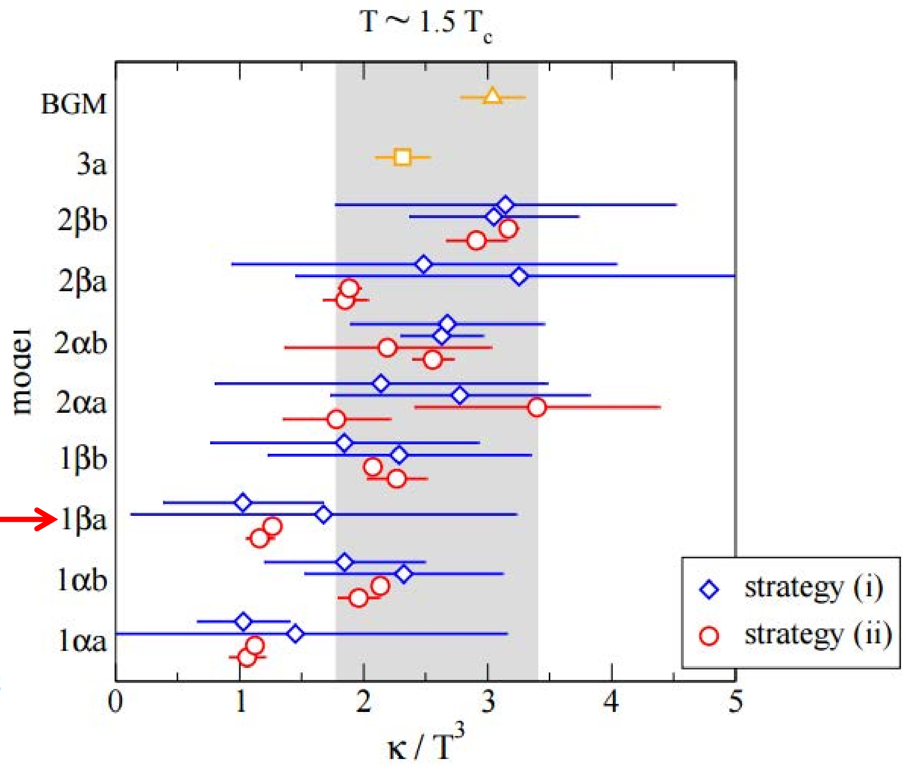
Correlator in the cont. limit



Perturbatively constrained model spectral functions



Heavy quark momentum diffusion coeff.



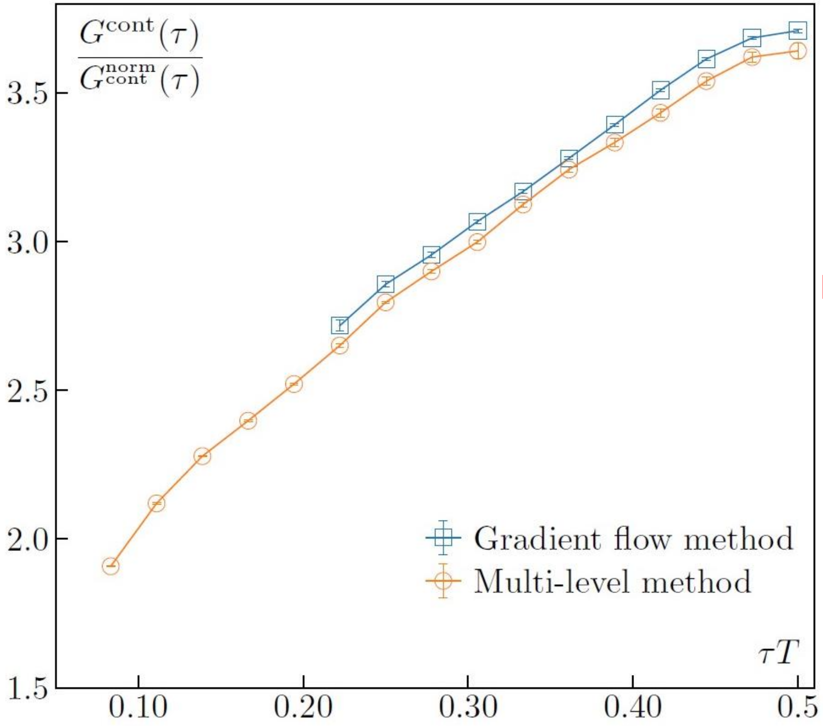
$$D = \frac{2T^2}{\kappa}$$ → **2πTD ≈ 3.7 — 7.0**

Heavy quark momentum diffusion coefficient: update

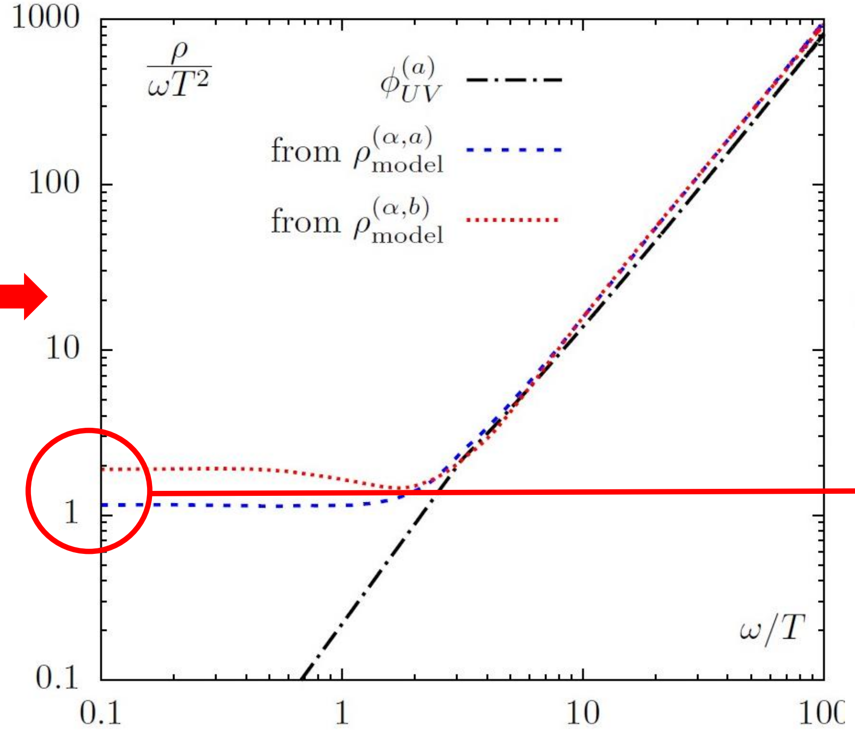
- Improvement with gradient flow

L. Altenkort, A. M. Eller, O. Kaczmarek, L. Mazur, G. D. Moore, H.-T. Shu, PRD 103 (2021) no.1, 014511

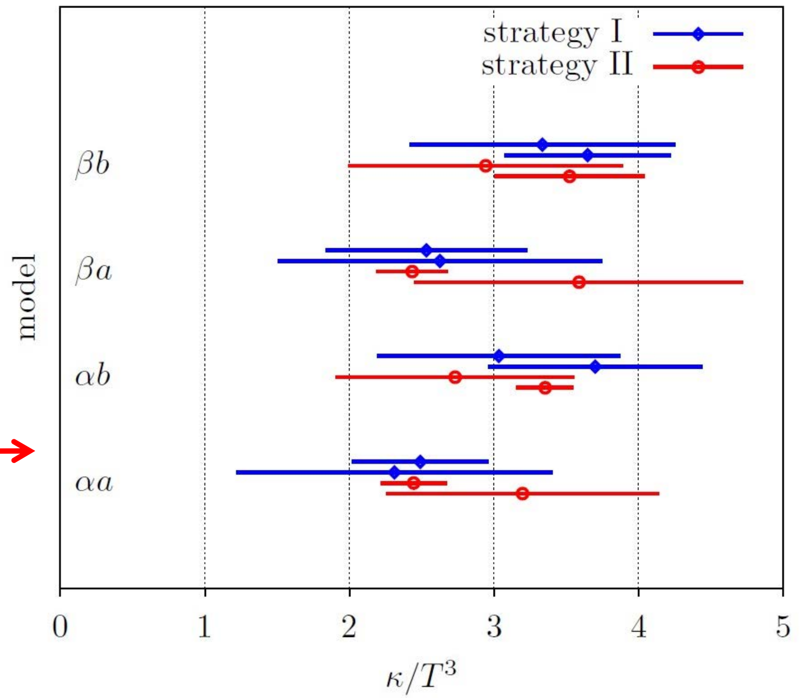
1.5T$_c$ Correlator in the cont. limit



Perturbatively constrained model spectral functions



Heavy quark momentum diffusion coeff.



$$D = \frac{2T^2}{\kappa} \rightarrow 2\pi TD \approx 1.7-2.7$$

Modeling quarkonium spectral functions in the pseudo-scaler channel

$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}}\theta(\omega-\omega^{match}) + A^{match}\underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}}\theta(\omega^{match}-\omega)\underbrace{\Phi(\omega)}_{\text{Suppression}}$$

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

- High energy ρ^{vac}: Vacuum asymptotics Burnier, Laine, Eur.Phys.J.C 72 (2012) 1902
- Threshold region ρ^{NRQCD}: pNRQCD Laine, JHEP 0705:028,2007
- Suppressed at low energy

Fitting lattice data to the model spectral function

- Quenched QCD, Clover Wilson, continuum extrapolated, pseudo-scalar

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

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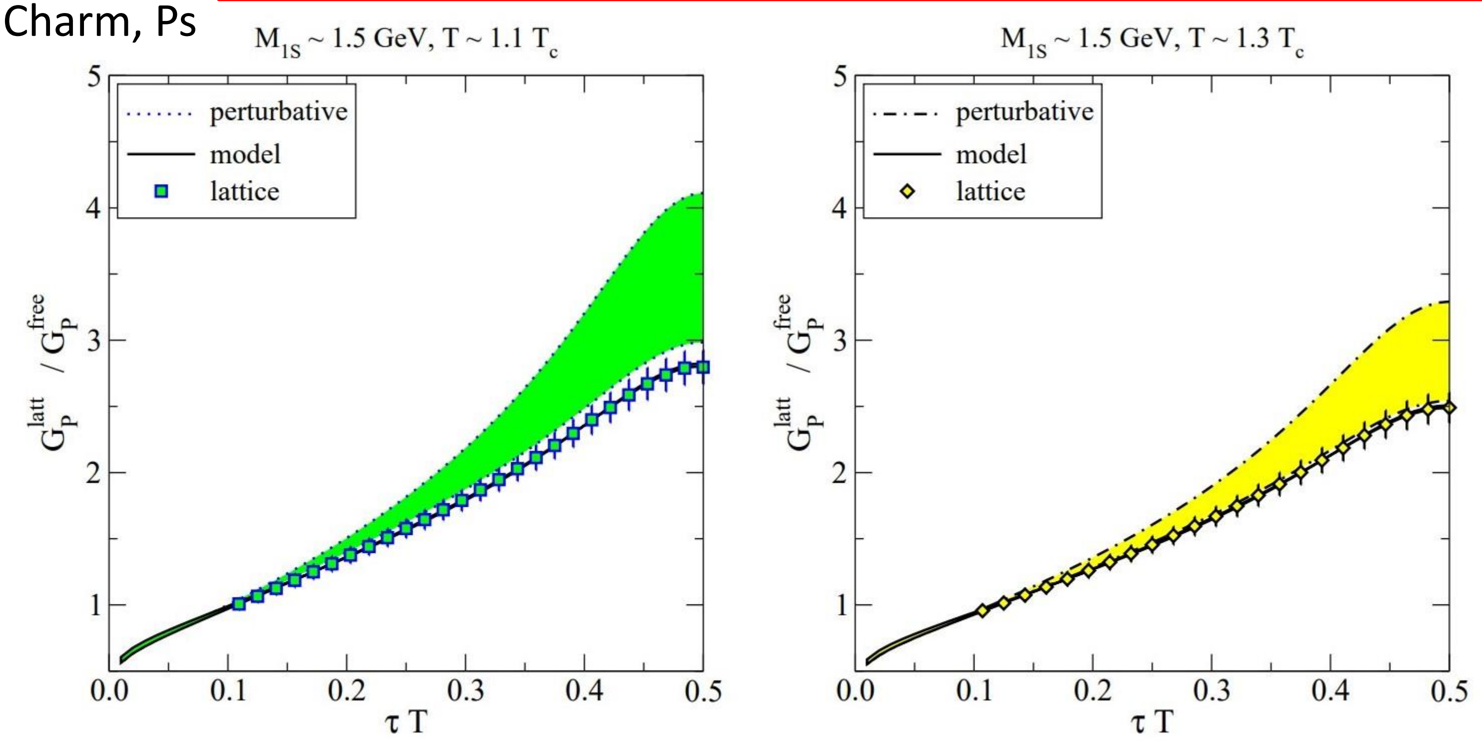
Charm, Ps



$$\rho^{\mathbf{mod}}(\omega) = \mathbf{A}\rho^{\mathbf{pert}}(\omega - \mathbf{B})$$

- **The model spectral function describes lattice data perfectly.**

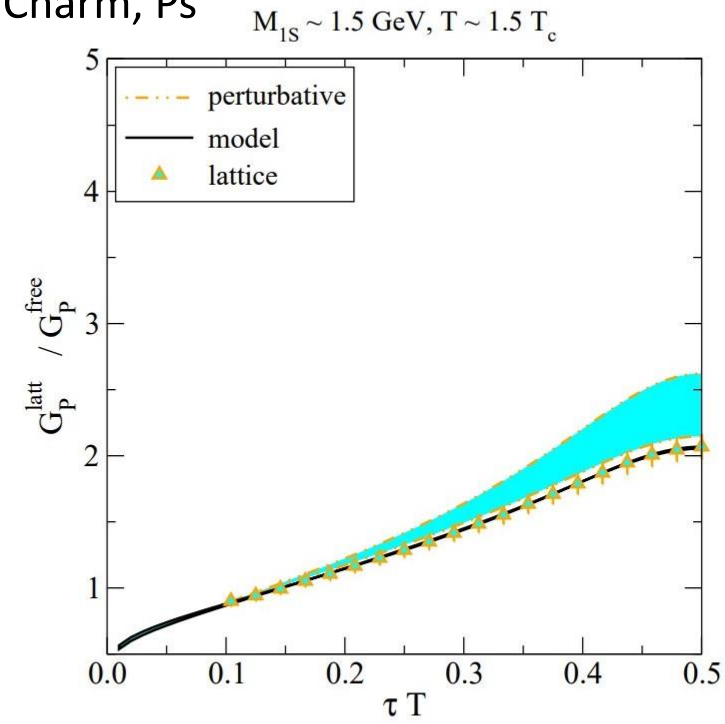
Fitting lattice data to the model spectral function

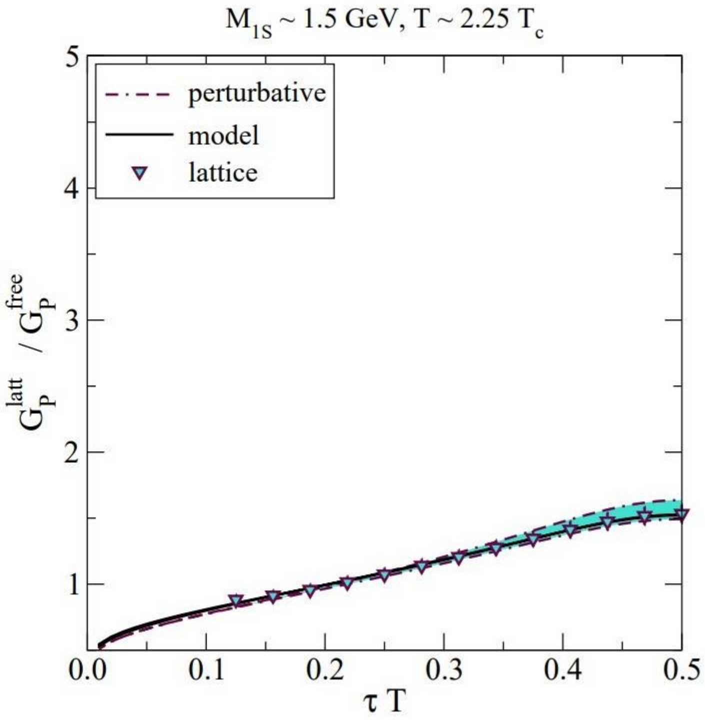
- Quenched QCD, Clover Wilson, continuum extrapolated, pseudo-scalar

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

$$\rho^{pert}(\omega) = \underbrace{\rho^{vac}(\omega)}_{\text{Vacuum asymptotics}} \theta(\omega - \omega^{match}) + A^{match} \underbrace{\rho^{NRQCD}(\omega)}_{\text{pNRQCD}} \theta(\omega^{match} - \omega) \underbrace{\Phi(\omega)}_{\text{Suppression}}$$

Charm, Ps





$$\rho^{mod}(\omega) = \mathbf{A}\rho^{pert}(\omega - \mathbf{B})$$

- **The model spectral function describes lattice data perfectly.**

Resulting spectral functions

- Quenched QCD, Clover Wilson, continuum extrapolated, pseudo-scalar

Y. Burnier, H. -T. Ding, O. Kaczmarek, A. -L. Kruse, M. Laine, HO, H. Sandmeyer, JHEP11(2017)206

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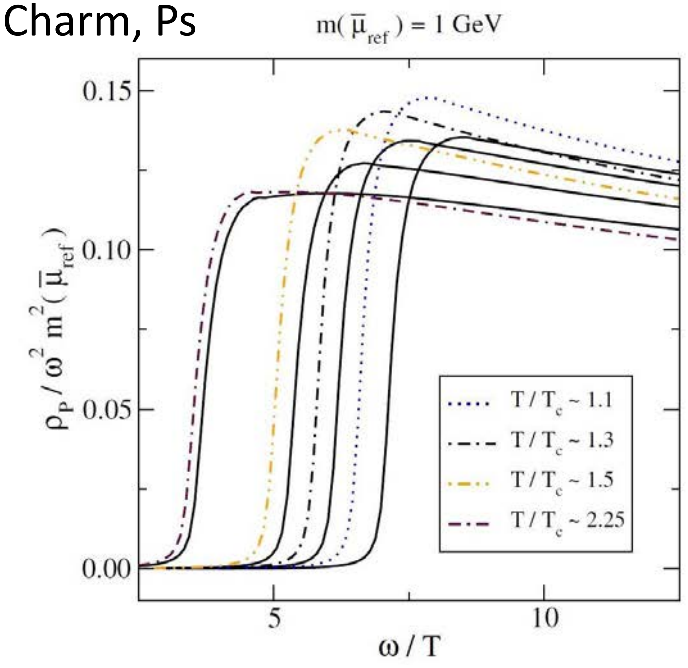
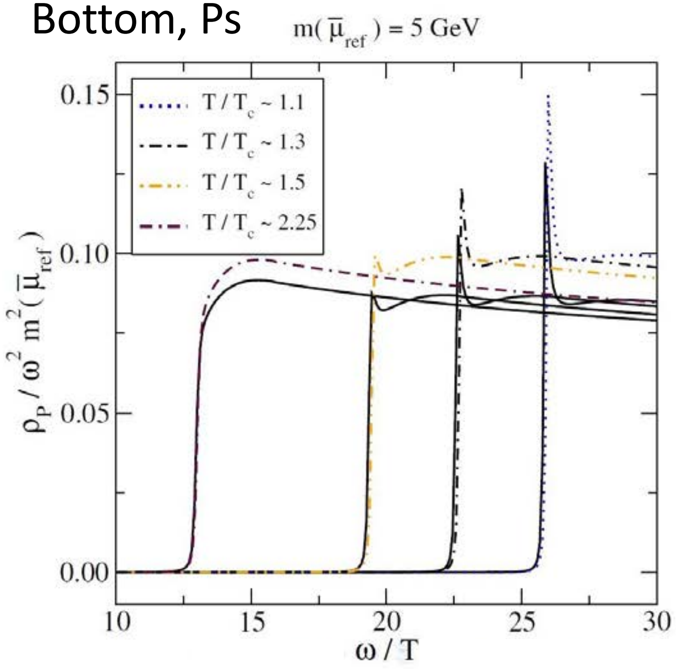
Charm, Ps



Bottom, Ps



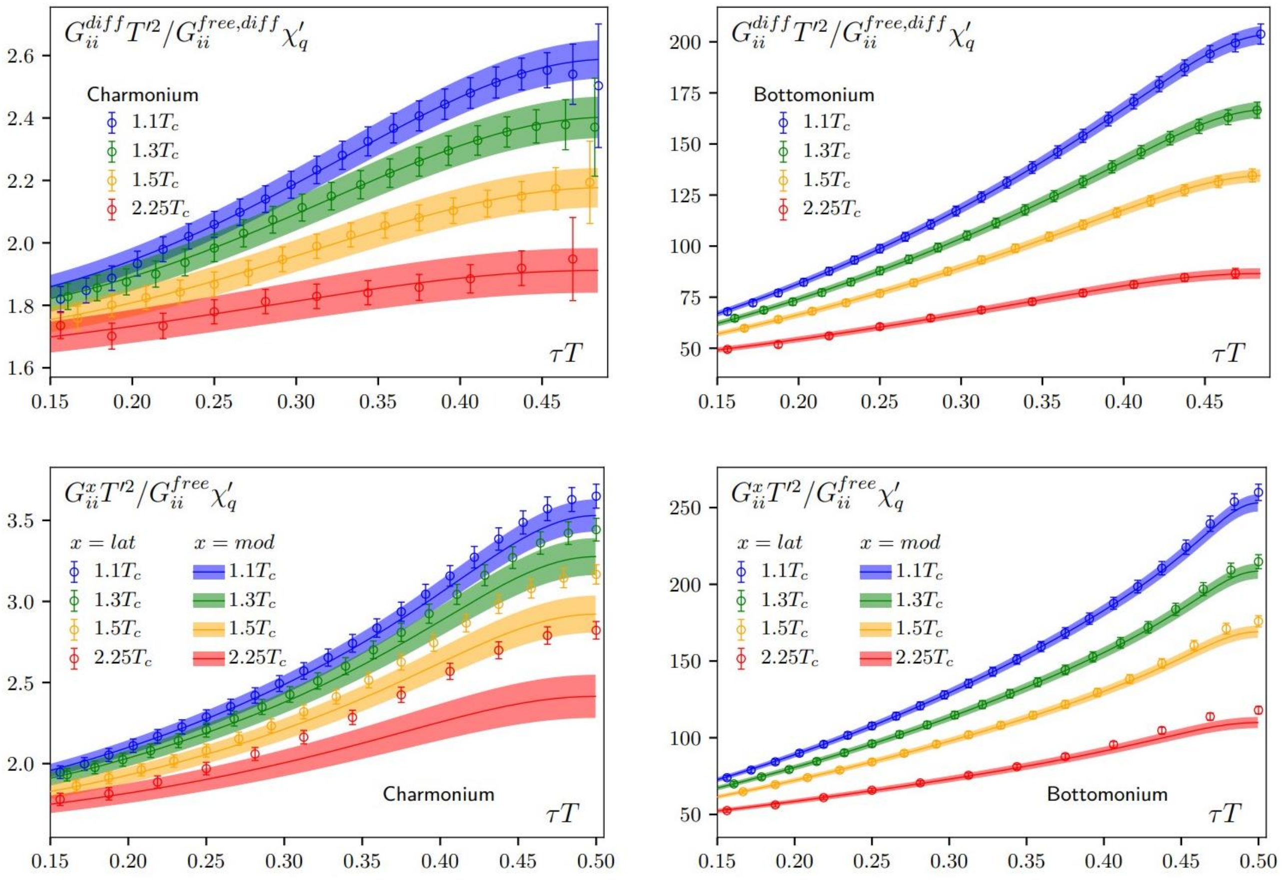
$$\rho^{\mathbf{mod}}(\omega) = \mathbf{A}\rho^{\mathbf{pert}}(\omega - \mathbf{B})$$

- **The model spectral function describes lattice data perfectly.**
- **No resonance peak needed for charm.**
- **A resonance peak is needed for bottom at T ≲ 1.5 Tc.**

Extension to the vector channel

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, arXiv:2108.13693 [hep-lat]



$$\rho^{\text{mod}}(\omega) = \mathbf{A}\rho^{\text{pert}}(\omega - \mathbf{B})$$

Transport peak is not described by ρ^{pert}

Transport peak : ω ~ 0
→ τ independent contributions
→ can be removed by correlator difference

$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a)$$

- **The model spectral function describes the correlator difference perfectly.**
- **Difference between the original lattice data and fit results**

→ Indication of the transport contributions

Resulting spectral functions

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

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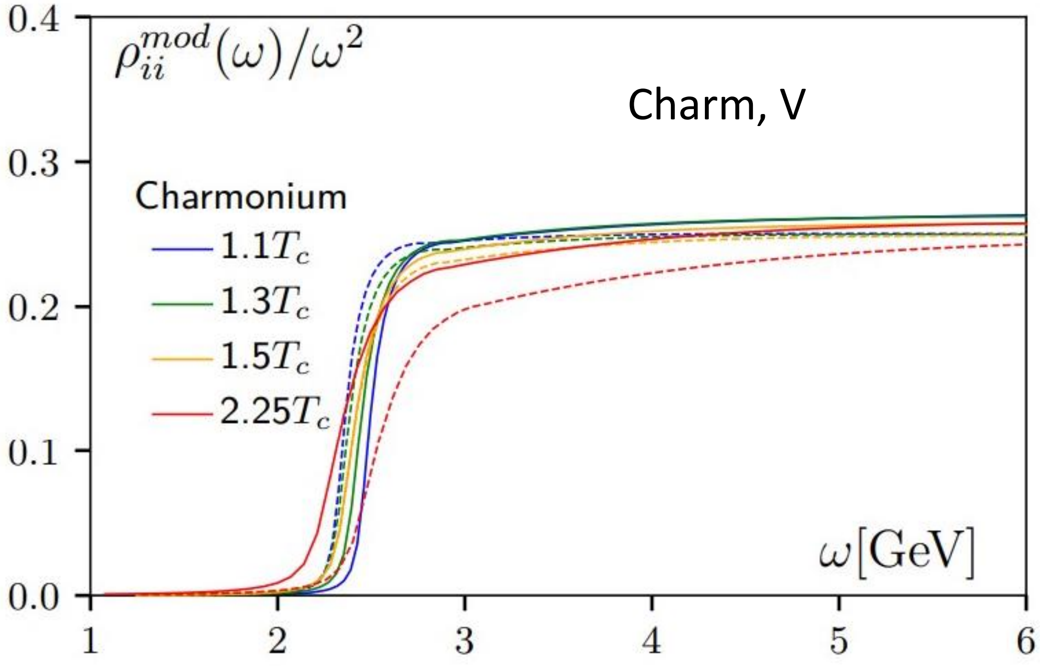

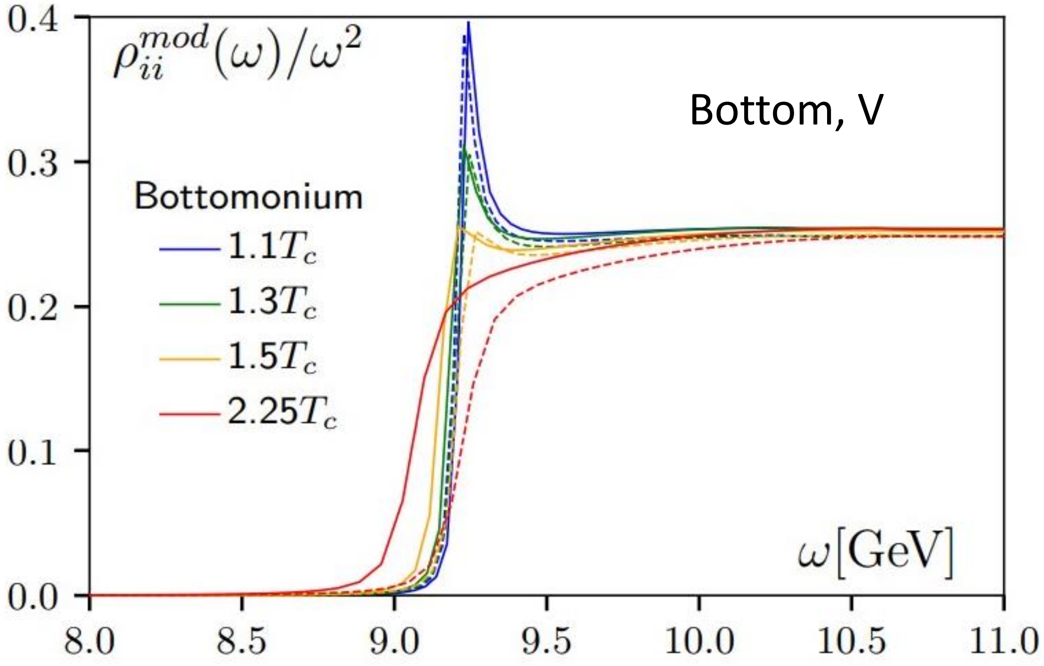

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Transport contributions

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

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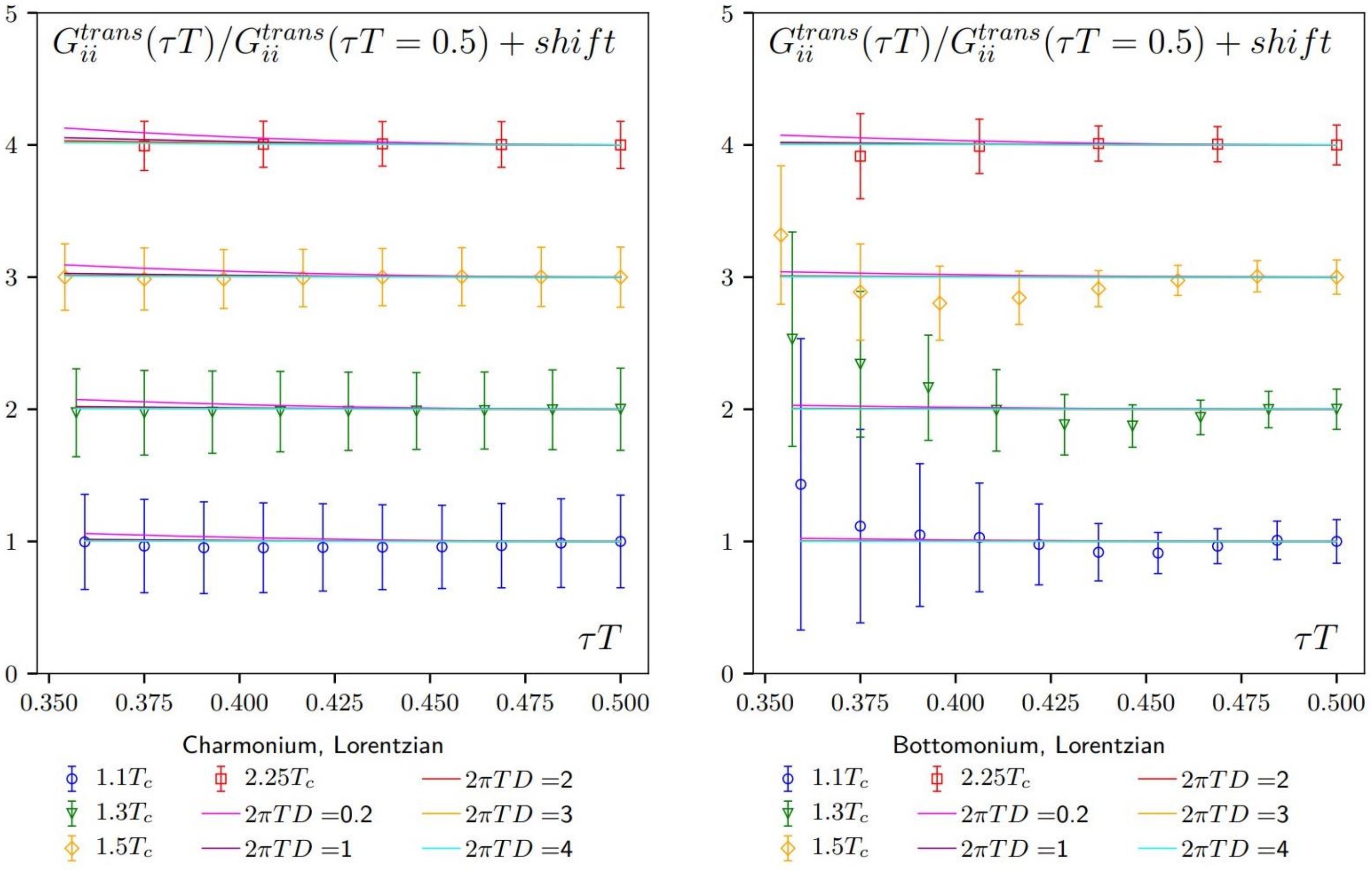


$$G_{ii}^{trans}(\tau T) = G_{ii}(\tau T) - G_{ii}^{mod}(\tau T).$$

$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}.$$

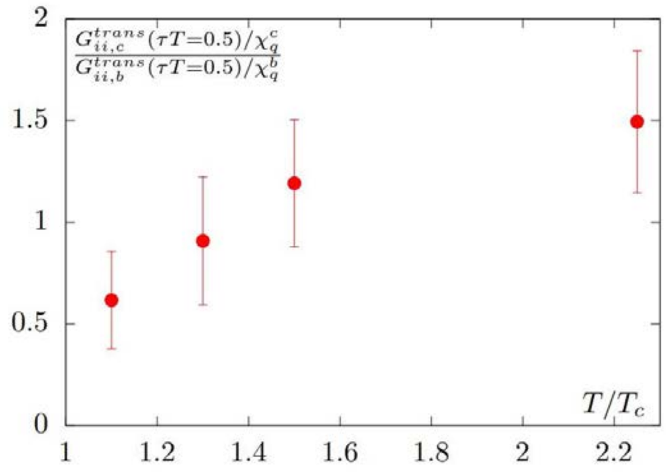
$$\eta = \frac{T}{MD}$$: Einstein relation

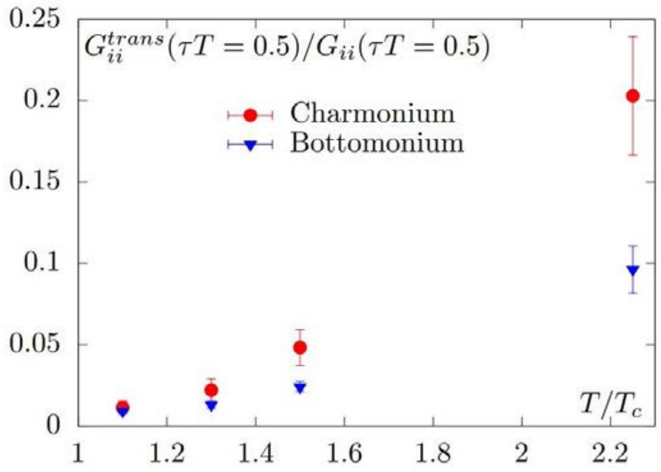
- **Hard to determine the transport coefficient D**

Transport contributions

- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, arXiv:2108.13693 [hep-lat]





$$G^{trans}_{ii}(\tau T) = G_{ii}(\tau T) - G^{mod}_{ii}(\tau T).$$

$$\rho^{trans}_{ii}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}.$$

$$\eta = \frac{T}{MD}$$: Einstein relation

$$\frac{G^{trans}_{ii,c}(\tau T = 0.5)/\chi^c_q}{G^{trans}_{ii,b}(\tau T = 0.5)/\chi^b_q} \approx \frac{M_b}{M_c}\frac{\arctan\left(\frac{\omega_{cut}}{\eta_c}\right)}{\arctan\left(\frac{\omega_{cut}}{\eta_b}\right)}.$$

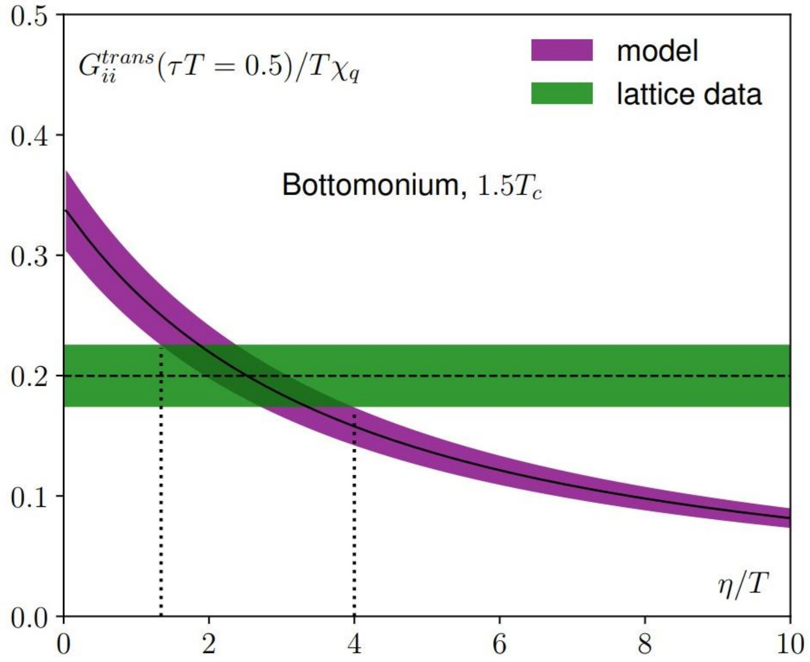
⇒ $\eta_c > \eta_b$

ω^{cut} : frequency to separate the transport and remaining parts

Estimation of the transport coefficient

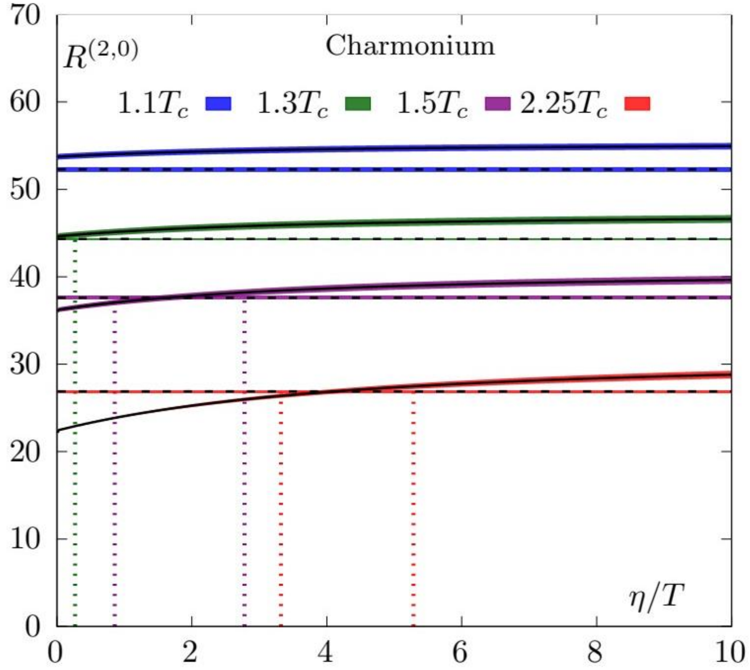
- Quenched QCD, Clover Wilson, continuum extrapolated, vector

H.-T. Ding, O. Kaczmarek, A.-L. Kurse, HO, H. Sandmeyer and H.-T. Shu, arXiv:2108.13693 [hep-lat]



$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}.$$

$$\eta = \frac{T}{MD}$$



Method 1: using correlator values at the midpoint

$$G_{ii}^{trans}(\tau T) = G_{ii}(\tau T) - G_{ii}^{mod}(\tau T).$$

Method 2: using the thermal moments of the correlator

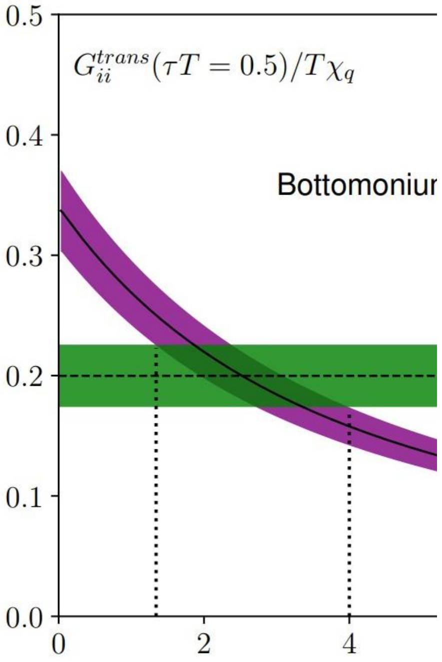
$$G_H^{(n)} = \frac{1}{n!}\int_0^\infty \frac{d\omega}{\pi}\left(\frac{\omega}{T}\right)^n \frac{\rho_H(\omega)}{\sinh(\frac{\omega}{2T})} \qquad G_H(\tau T) \approx G_H^{(0)} + G_H^{(2)}(\tau T-0.5)^2 + \cdots$$

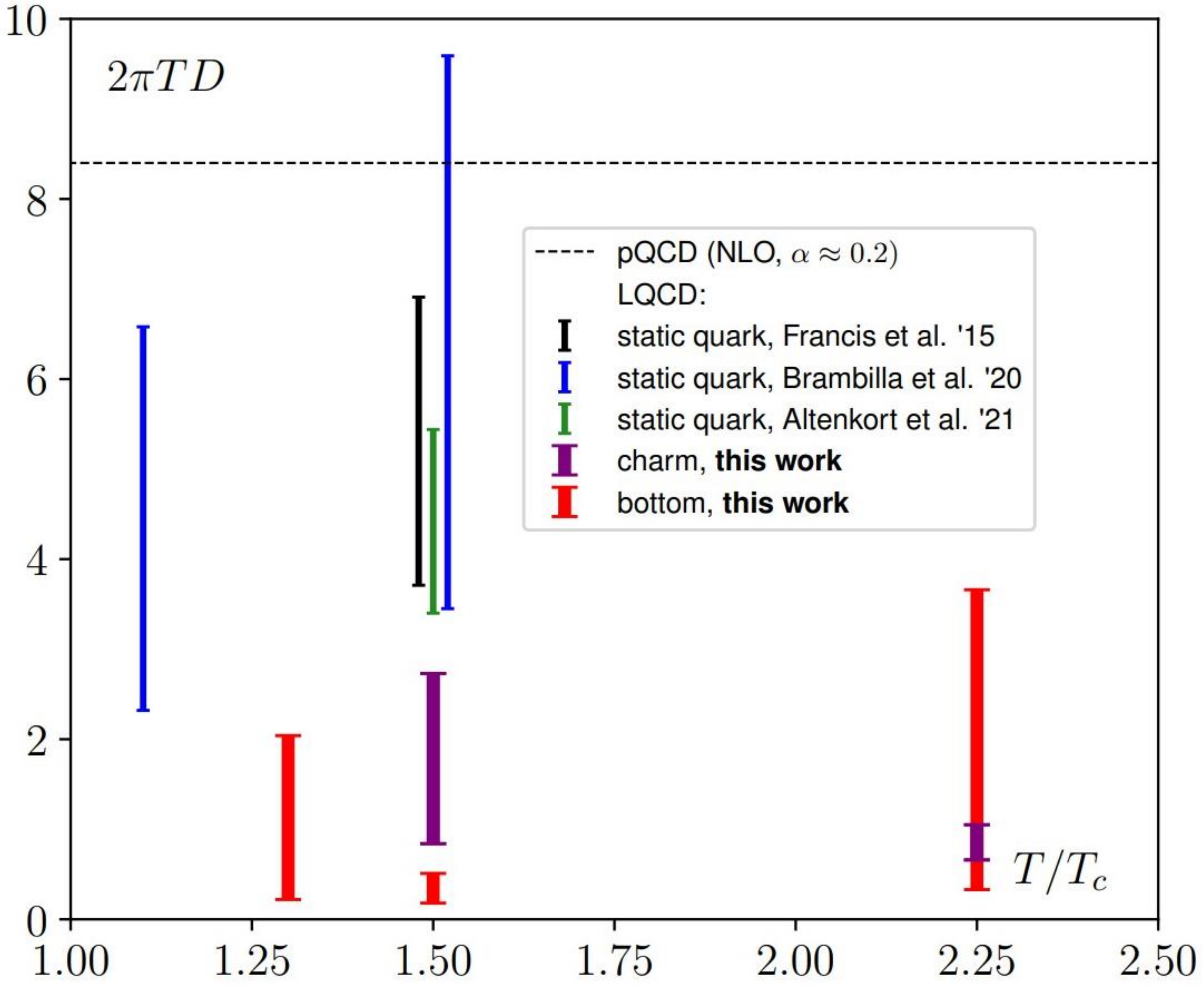
$$R_H^{n,m} = \frac{G_H^{(n)}}{G_H^{(m)}} \qquad \frac{\Delta_H(\tau T)}{G_H(\tau T=0.5)} \approx R_H^{2,0} \qquad \Delta_H(\tau T) = \frac{G_H(\tau T) - G_H(\tau T=0.5)}{(\tau T-0.5)^2}$$

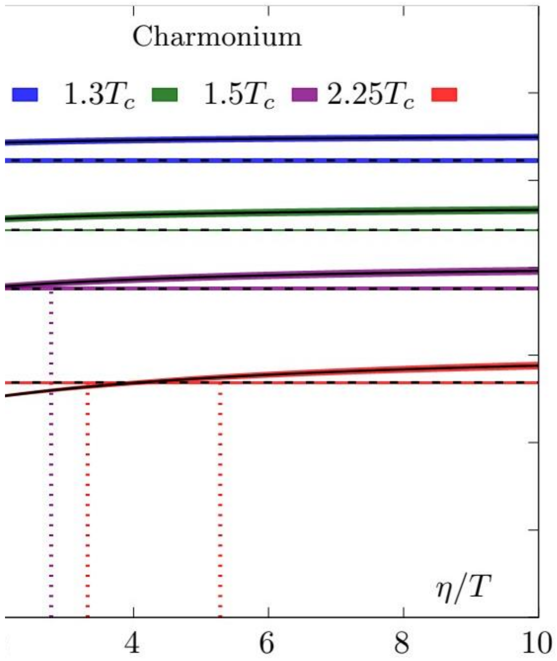
Estimation of the transport coefficient

- Quenched , vector

H.-T. Ding, hep-lat]







Method 1: using co mal moments of the correlator

$$G_{ii}^{trans}(\tau T) = G_i$$

$$G_H(\tau T) \approx G_H^{(0)} + G_H^{(2)}(\tau T - 0.5)^2 + \cdots$$

$$\approx R_H^{2,0} \quad \Delta_H(\tau T) = \frac{G_H(\tau T) - G_H(\tau T = 0.5)}{(\tau T - 0.5)^2}$$

Summary and outlook

- Heavy quarks are important probes to investigate quark-gluon-plasma.
- Spectral functions contain all information about in-medium properties of heavy quarks.
- There are some difficulties to study the spectral functions on the lattice.
- Heavy quark momentum diffusion coefficient was estimated by fitting the color electric correlator to a perturbative model spectral function.
- Charmonium and bottomonium spectral functions were determined from continuum extrapolated quenched lattice correlators in pseudo-scalar and vector channels
- Heavy quark diffusion coefficient was also estimated.
- Extension to full QCD is a future work.