# The physics of neutrinos

## ~Dirac vs. Majorana~

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# 1． Key roles played by neutrinos

・P violation in weak interactions : $\nu_L$ $\cancel{\nu_R}$ in SM （Standard Model)

・definitive evidence of BSM (beyond standard model) :

neutrino oscillations ⇒ massive neutrinos

・ unsolved problem in SM: matter-antimatter asymmetry in the universe

← heavy Majorana neutrino ``leptogenesis” （Fukugita & Yanagida )

・ cosmic neutrino background (C $\nu$ B) of 1.9 K ( $T_\nu = \left(\frac{2}{2+\frac{7}{8}(2+2)}\right)^{\frac{1}{3}} T_\gamma$ )

very good probe of very early universe (origin of our universe)

Cosmic Background Neutrino Decay Search （COBAND） exp.

# 2. Dirac vs. Majorana

- Charged fermion： Dirac fermion

  described by 2 × 2 = 4 component complex Dirac spinor $\psi_D$

- Is there fermion without discrimination between particle –antiparticle ?

  Possible only for neutral fermions, like neutrino (photino, ..)： Majorana fermion

  described by Majorana spinor $\psi_M$ ： $\psi_M^c = \psi_M$ $(c: \ \text{charge congugation})$

- Take scalar particle (s = 0) . Charged scalar is described by a complex field $\phi$.

  (gauge transformation) $\phi \ \rightarrow \ \phi' = e^{iq\lambda}\phi \ (q: \ \text{electric charge})$

  ⇒ $\phi$ ： complex

$$\phi^* \ \rightarrow \ \phi'^* = e^{i(-q)\lambda}\phi^*$$

：charge conjugation = complex conjugation

- A scalar without discrimination between particle –antiparticle (e.g. Higgs):

$$\phi^* = \phi \quad \rightarrow \quad \phi : \mathrm{real}$$

- Similarly Majorana spinor is 4-component real spinor.

Majorara is more fundamental than Dirac.

$$z = a + bi \quad (a, b : \ \mathrm{real})$$

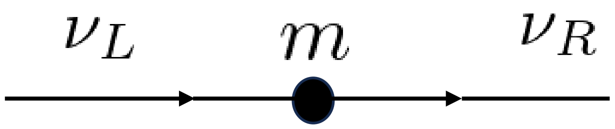
- Similarly, a Dirac spinor is equivalent to 2 Majorana spinors with identical masses:

$$\psi = \frac{\psi_1 + i\psi_2}{\sqrt{2}} \quad (\psi_1 = \frac{\psi + \psi^c}{\sqrt{2}}, \psi_2 = -i\frac{\psi - \psi^c}{\sqrt{2}}) \quad ((\psi^c)^c = \psi)$$

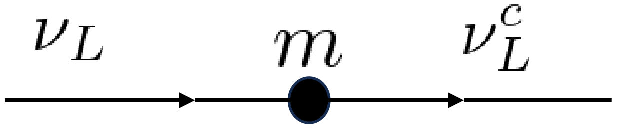
・If neutrinos are Majorana, lepton number L ： indefinite

``Majorana mass term” causes L violation.

In general, mass term causes chirality-flip and necessitates ``chiral partner”



Dirac (active → ``sterile”)



Majorana (active → active, $|\Delta L| = 2$)

# 3. Neutrino oscillation

A sort of lepton-flavor changing process :

e.g. $\nu_e \to \nu_\mu$

Assume 2 generation scheme

$$\nu_e = \cos\theta\ \nu_1 + \sin\theta\ \nu_2$$
$$\nu_\mu = -\sin\theta\ \nu_1 + \cos\theta\ \nu_2 \quad (\nu_{1,2} :\ \text{mass eigenstates})$$

Transition probability at time t :

$$P(\nu_e \to \nu_\mu) = |\cos\theta\sin\theta(e^{iE_2t} - e^{iE_1t})|^2 = \sin^2(2\theta)\sin^2(\frac{\Delta m^2}{4E}t)$$
$$(\ E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2E} \text{ for } p \gg m_i\ (i = 1, 2),\ \Delta m^2 = m_2^2 - m_1^2\ )$$

The physics is basically the same as ``beat”

$$\begin{tikzcd}
 & \nu_1 \arrow[r, "e^{-iE_1t}"] & \nu_1(t) \arrow[dr, "-\sin\theta"] & \\
\nu_e \arrow[ur, "\cos\theta"] \arrow[dr, "\sin\theta"'] & & & \nu_\mu \\
 & \nu_2 \arrow[r, "e^{-iE_2t}"] & \nu_2(t) \arrow[ur, "\cos\theta"'] &
\end{tikzcd}$$

For neutrino oscillation,

- $\theta \neq 0$ $\quad \cdot\, \Delta m^2 = m_2^2 - m_1^2 \neq 0$ ⇒ massive neutrinos

## 4. Three types of neutrino

Neutrino oscillation ⇒ massive neutrinos ⇒ BSM

We minimally extend SM by introducing $\nu_R$

The most general mass term for neutrino (1 generation):

$$\begin{aligned} -\mathcal{L}_{\rm mass} &= m_D\ \bar{\nu}_R \nu_L + \frac{1}{2} m_R\ \bar{\nu}_R^c \nu_R + \text{h.c.} \\ &= \frac{1}{2} \begin{pmatrix} \bar{\nu}_L^c & \bar{\nu}_R \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix} + \text{h.c.} \end{aligned}$$

(a) Dirac ($m_R = 0$)

mass eigenvalues: $m_D$ (degenerated)

mass eigenstates: $\frac{\nu_L + \nu_R^c}{\sqrt{2}} \; (= \nu_{sL})$ $\quad i\frac{\nu_L - \nu_R^c}{\sqrt{2}} \; (= \nu_{aL})$ $\quad \nu_s, \; \nu_a$ : Majorana

1 Dirac = 2 Majorana with mass degeneracy

mixing angle: $\theta = \frac{\pi}{4} \; : \; \text{maximal mixing}$ (between active & sterile)

(N.B.) Though mixing is maximal, we cannot expect neutrino oscillation (active ⇔ sterile) because of mass degeneracy.

## (b) Seesaw ( $m_D << m_R$ )

- $\theta \simeq 0$

- mass-eigenstates: $i(\nu_L - \nu_L^c), \quad \nu_R + \nu_R^c$

- mass-eigenvalues: $\frac{m_D^2}{m_R} \; (<< m_D), \quad m_R$

⇒ smallness of neutrino masses is naturally explained

@ low energies, only 1 light Majorana neutrino

message: no neutrino oscillation in 1 generation scheme for Dirac or Seesaw

## (c) pseudo-Dirac ( $m_D >> m_R$ )

(Wolfenstein, ···)

- almost Dirac & $\theta \simeq \frac{\pi}{4}$ : almost maximal
- But, the degeneracy of mass eigenvalues is slightly lifted: $\delta m^2 \simeq 2 m_D m_R$

two independent light Majorana neutrinos for each generation

⇒ neutrino oscillation (active ⇔ sterile) with maximal mixing, even in 1 generation scheme !!

( M. Kobayashi & C.S.L., PRD 64 ('01)013003;
see also M. Kobayashi, C.S.L. & M.M. Nojiri, PRL 67('91)1685 )

# 5. Neutrino oscillations and Majorana nature of neutrino

How can we reveal the Majorana nature ?

- Promising terrestrial experiment: 0 ν β β $(Z, A) \to (Z+2, A) + 2e^-$ ( $\Delta L = 2$ )

  (KamLAND-ZEN, GERDA, PIKACHU, ⋯)

  challenging, the decay width being suppressed by small neutrino mas (- squared).

- How about neutrino oscillation ?

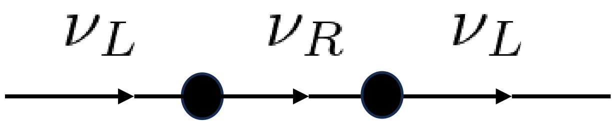
  Unfortunately, Dirac and Seesaw provide identical predictions (cannot be discriminated)

  since ,

  (i) chirality-flipping oscillation, $\nu_L \to \nu_R$ (for Dirac), $\nu_L \to \nu_L^c$ (for seesaw)

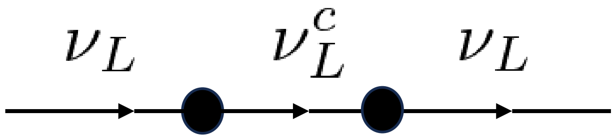
  are strongly suppressed by $\left(\frac{m_\nu}{E}\right)^2$

(ii) chirality-preserving oscillation, $\nu_L \to \nu_L$, cannot discriminate two cases:



(Dirac)

$(\Delta L = 0)$



(Seesaw)

Majorana nature cannot be revealed, as long as we take seesaw as correct scenario for Majorana neutrino.

In both of Dirac & seesaw scenarios, the unitarity among active states holds:

$$\sum_\beta P(\nu_{\alpha L} \to \nu_{\beta L}) = 1 \quad (\alpha, \beta = e, \mu, \tau)$$

# 6． What we have learned from the neutrino oscillation

(Chirality preserving) ordinary flavor oscillations do not discriminate Dirac & Seesaw

⇒ described by 3 × 3 matrix (PMNS) with 3 mixing angles $\theta_{12},\ \theta_{23},\ \theta_{13}$

and CP violating phase $\delta$ , together with 2 mass-squared differences,

$$\Delta m^2_{21},\ \Delta m^2_{32}\ \ (\Delta m^2_{ij} \equiv m^2_i - m^2_j)$$

【remaining issues】

・ mass ordering (normal or inverted) : the sign of $\Delta m^2_{32}$

・ CP violation in leptonic sector: $J = \cos\theta_{12}\sin\theta_{12}\cos\theta_{23}\sin\theta_{23}\cos^2\theta_{13}\sin\theta_{13}\sin\delta$

T2HK (T2KK), LBNF/DUNE

(NB) Majorana CP phase, important in leptogenesis, is not attainable.

# 7. Two possible ways to reveal Majorana nature (relying on neutrino oscillation)

## (1) Neutrino oscillation of pseudo-Dirac neutrinos

Oscillation without chirality flip : $\nu_L \to \nu_R^c \quad (\text{active} \to \text{sterile}, \ L\text{−violating})$
, which makes difference:

$$\sum_{\beta} P(\nu_{\alpha L} \to \nu_{\beta L}) < 1$$

If oscillations of this type are found, it will be a clear signal of the pseudo-Dirac neutrinos
, and therefore Majorana nature of neutrinos

This scenario is disfavored as the solution of solar or atmospheric neutrino deficit.

(e.g. from Super-Kamiokande & SNO for solar $\nu$ )

→ $\delta m^2 \leq 10^{-11}\ \mathrm{eV}^2$ (from solar $\nu$ )

However、 we have a chance to detect the oscillation of pseudo Dirac neutrinos by use of neutrinos with astrophysical or cosmic origin utilizing their very long baseline.

- Flavor composition of ultra-high energy astrophysical neutrino flux

  (detected e.g. by Ice-Cube)

  is distorted by this pseudo-Dirac type oscillation

(J.F. Beacon, N.F. Bell, D. Hooper, J.G. Learned, S. Pakvasa, T.J. Weiler, PRL 92('04)011101)

Ordinary flavor oscillation: $\phi_e : \phi_\mu : \phi_\tau = 1 : 1 : 1$

Taking into account pseudo-Dirac type oscillation, it can be e.g.

$\phi_e : \phi_\mu : \phi_\tau = 4 : 3 : 3$ (if largest $\delta m^2$ is large enough )

This is based on a formula

$$\phi_\alpha = \frac{1}{3}\sum_i |U_{\alpha i}|^2\left\{1-\sin^2\left(\frac{\delta m_i^2}{4E}L\right)\right\} \quad (U:\text{PMNS matrix},\ L:\text{baseline})$$

## 【 Formalism 】

( M. Kobayashi & C.S.L., PRD 64 ('01)013003 )

We have 2 × 3 = 6 light neutrinos ⇒ mass matrix: 6×6 : terrible !

However, invoking $m_D \gg m_R$

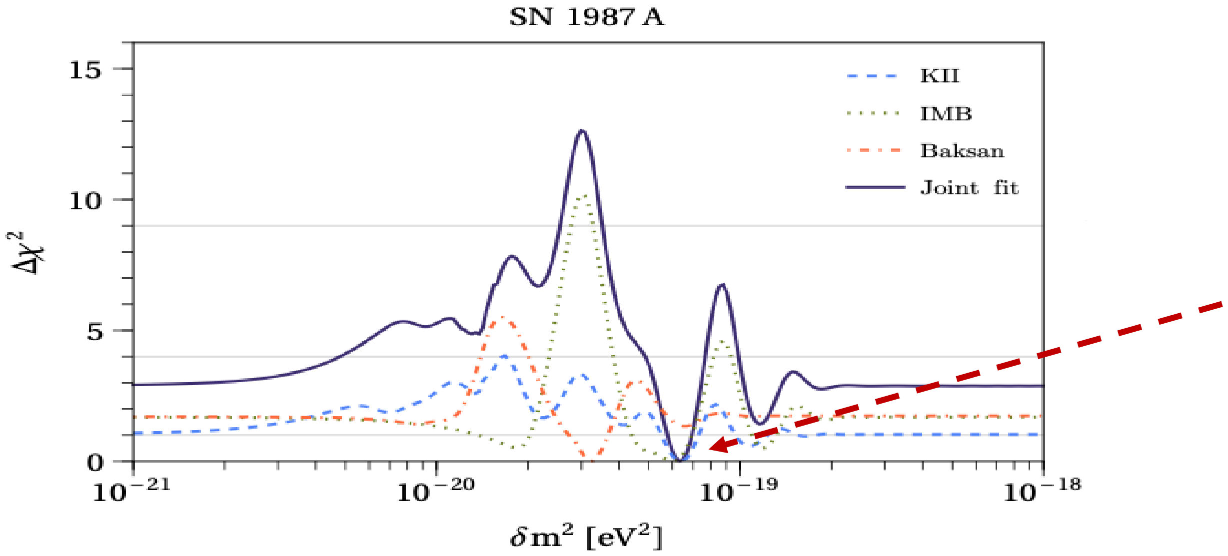
- diagonalize the Dirac mass matrix by use of PMNS matrix
- there appear degeneracies in the mass eigenvalues for each generation
- perform exact diagonalization in each of 2×2 sub-matrix

to get $\nu_{\alpha L} = U_{\alpha j}\dfrac{\nu_{sj}+i\nu_{aj}}{\sqrt{2}} \quad (\alpha = e, \mu, \tau)$

# (Recent discussion in the literature)

・A possible sign in SN1987A data（K II , IMB, Baksan）？

(I. Martinez-Soler, Y.F. Perez-Gonzalez, M. Sen, Phys.Rev.D 105 (2022) 9, 095019)



mild preference for
$\delta m^2 = 6.31 \times 10^{-20}\ \mathrm{eV}^2$

・Diffuse supernova neutrino background（Y.F. Perez-Gonzalez,　CERN Neutrino Platform ’23）

← SK-Gd, HK,　DUNE

・Capture rate of C $\nu$ B（PTOLEMY exp.）:　sensitive to $\delta m^2 \sim 10^{-35}\ \mathrm{eV}^2$

（Y.F. Perez-Gonzalez, M. Sen, 2308.05147[hep-ph]）

## (2)　RSFP　(Resonant Spin-Flavor Precession)

( C.S.L. & W.J. Marciano, PRD37 ('88)1368;　E.Kh. Akhmedov, PL B213('88)64 )

Neutrino oscillation with chirality-flip, but without suppression factor,
(thus making discrimination between Dirac and Majorana possible):

spin precession under strong external magnetic field (the interior of the Sun, supernovae etc.), whose probability

$$P(\nu_L \to \nu_R) = \sin^2(\mu B t) \qquad \text{(in the vacuum)}$$

: nothing to do with neutrino masses

- Originally, RSFP was proposed to solve the solar neutrino problem.

Though ordinary precession $\nu_{eL} \to \nu_{eR}$ is suppressed by ``matter effect" ,
if we consider spin-flavor precession such as

$$\nu_{eL} \to \nu_{\mu R}$$

the matter effect leads to resonant enhancement (a la MSW) of the precession.

- For Majorana neutrino, RSFP is L-violating (active →active) oscillation, such as

$$\nu_{eL} \to (\nu_{\mu L})^c$$

Interaction term relevant for this process:

$$\mu_{\alpha\beta}\ \overline{(\nu_{\alpha L})^c}\sigma_{\mu\nu}\nu_{\beta L} \cdot F^{\mu\nu} \quad (\alpha, \beta = e, \mu, \tau)$$

where

$$\mu_{\alpha\beta} = -\mu_{\beta\alpha} \quad \to \quad \mu_{\alpha\alpha} = 0$$

⇒ for Majorana neutrino the transition magnetic moment $\mu_{\alpha\beta}\ (\alpha \neq \beta)$

is inevitable

- Radiative decay of neutrinos $\nu_i \to \nu_j + \gamma$ utilized in COBAND exp. to search for C ν B is exactly due to the transition magnetic moment

$$\mu_{ji}\ \overline{\nu_j}\sigma_{\mu\nu}\nu_i \cdot F^{\mu\nu}\ \ (i \neq j)$$

⇒ RSFP & COBAND both need BSM to realize ``sizable” neutrino magnetic moment, while keeping neutrino masses small

# (Recent discussion in the literature)

- For Majorana  neutrinos, RSFP & flavor mixing will imply $\bar{\nu}_e$ from the Sun
  ( C.S.L., M. Mori, Y. Oyama, A. Suzuki, Phys.Lett.B 243 ('90) 389 ;
  E. Akhmedov, P. Martinez-Mirave, JHEP10 ('22)144 )
  it would be clear signal of Majorana nature (L violating $\nu_e \rightarrow \bar{\nu}_e$ )

- $\nu_e \rightarrow \bar{\nu}_e$ due to RSFP is also expected for supernova neutrino produced by neutronization burst, and may be tested by HK & DUNE
  (S. Jana, Y.P. Porto-Silva, M. Sen, ICHEP 2022)