On the Efficiency Correction for non-Binomial Efficiency Loss

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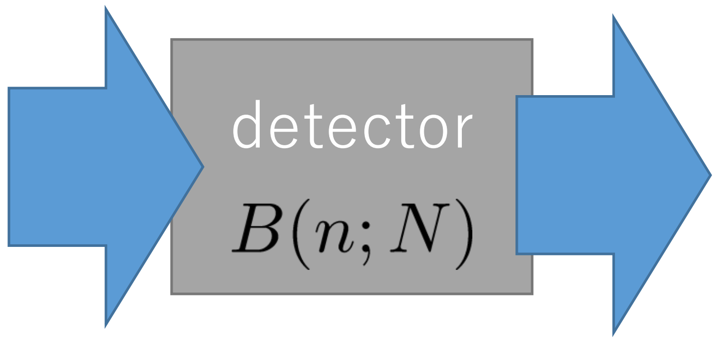
2017.12.11, Tsukuba U.

Efficiency Correction

N injected particles
$P(N)$



n observed particles
$\tilde{P}(n)$

$$\tilde{P}(n) = \sum_{N} P(N) B(n;N)$$

Efficiency correction problem:
Construct cumulants of N from distribution of n.

Bialas, Peschanski, 1986
MK, Asakawa, 2012;
Bzdak, Koch, 2012, 2015;
MK, 2016;
Nonaka, MK, Esumi, 2017

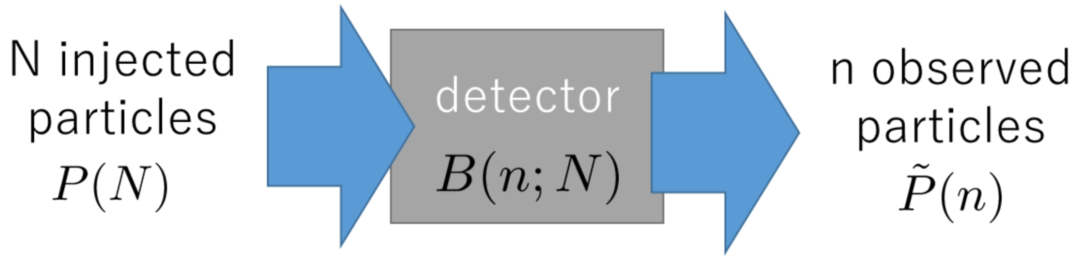
Binomial Model

$$\tilde{P}(n) = \sum_{N} P(N) B(n; N)$$

B: binomial distribution funciton

- Analytic formulas for efficiency correction
- The model is valid when efficiency is independent for individual particles.
- Not perfectly applicable to real detectors.
- Estimate on non-binomial correction would be crucial for estimating systematic errors reliably.

Why Binomial Model so Useful?



- Simple relation b/w factorial cumulants

$$\langle n^m \rangle_{\mathrm{fc}} = p^m \langle N^m \rangle_{\mathrm{fc}}$$

→ quite useful in reducing numerical cost for many efficiency bins

Nonaka, MK, Esumi, 2017

- Cumulants of n are proportional to N

$$\langle n^m \rangle_{N,\mathrm{c}} = \xi_m N$$

↑ cumulant of binomial (with fixed B)

→ Short efficiency correction formula can be obtained (with a hard algebra)

MK, 2016

How to deal with non-binomial efficiency?

- Unfolding methods
  - large phase space / large numerical cost / estimate on convergence, systematics …
  - Can we perform unfolding only by cumulants?
- Use specific distributions?
  - hypergeometric func., etc.
  - no a priori justification

My Suggestion 1: New Formula

$$\tilde{P}(n) = \sum_N P(N) B(n;N)$$

When B(n;N) satisfies the following property

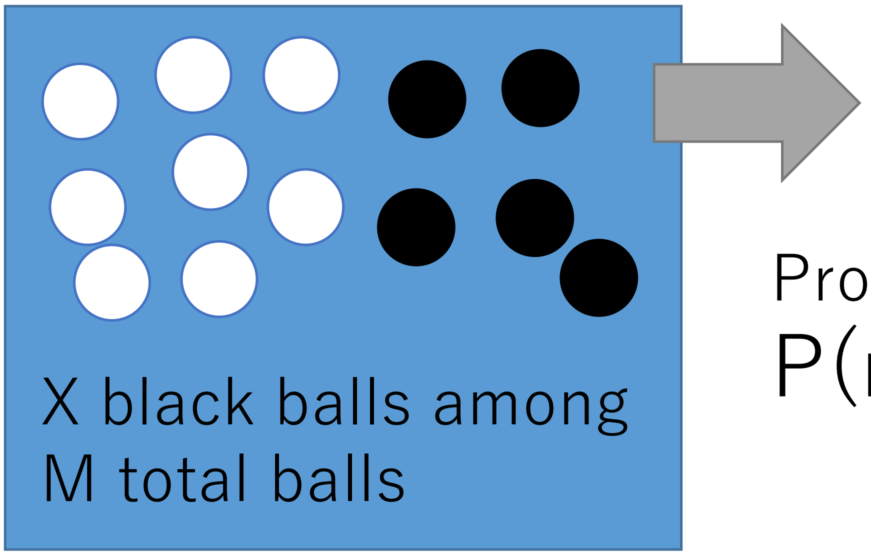
$$\langle n^m \rangle_{N,\mathrm{c}} = \sum_{i=0}^{m} c_{mi} N^i$$

cumulant of B

$$\langle n \rangle_N = c_{10} + c_{11} N$$
$$\langle n^2 \rangle_{N,\mathrm{c}} = c_{20} + c_{21} N + c_{22} N^2$$
$$\langle n^3 \rangle_{N,\mathrm{c}} = c_{30} + c_{31} N + c_{32} N^2 + c_{33} N^3$$

- We can obtain efficiency correction formula!
- (Cumulants of N can be represented by cumulants of n)
- Costs are the same order as MK2016.

Example: Hypergeometric Dist.



draw N balls

Probability to find n black balls

P(n;N,X,M)

$$\langle n \rangle_N = pN$$

$$\langle n^2 \rangle_{N,\mathrm{c}} = p(1-p)N\left(1 - \frac{N-1}{X-1}\right)$$

Is this distribution a proxy of experimental detectors??

Formulas: Single Variable Case

$$\langle n\rangle_{\rm c} = c_{10} + c_{11}\langle N\rangle_{\rm c}$$

$$\langle n^2\rangle_{\rm c} = c_{20} + c_{10}^2 + (c_{21} + 2c_{10}c11)\langle N\rangle_{\rm c} + (c_{22} + c_{11}^2)\langle N^2\rangle_{\rm c} + c_{22}\langle N\rangle^2$$

$$\langle n^3\rangle_{\rm c} = C_0 + C_1\langle N\rangle_{\rm c} + C_2\langle N^2\rangle_{\rm c} + C_3\langle N^3\rangle_{\rm c} + D_1\langle N^2\rangle_{\rm c}\langle N\rangle_{\rm c} + D_2\langle N^3\rangle_{\rm c}$$

$$C_0 = c_{30} + 3c_{10}c_{20}$$

$$C_1 = c_{31} + 3c_{10}c_{21} + 3c_{11}c_{20} + 3c_{10}^2c_{11}$$

$$C_2 = c_{32} + 3c_{11}c_{21} + 3c_{10}c_{11}^2$$

$$\langle n\rangle_N = c_{10} + c_{11}N$$

$$\langle n^2\rangle_{N,\rm c} = c_{20} + c_{21}N + c_{22}N^2$$

$$\langle n^3\rangle_{N,\rm c} = c_{30} + c_{31}N + c_{32}N^2 + c_{33}N^3$$

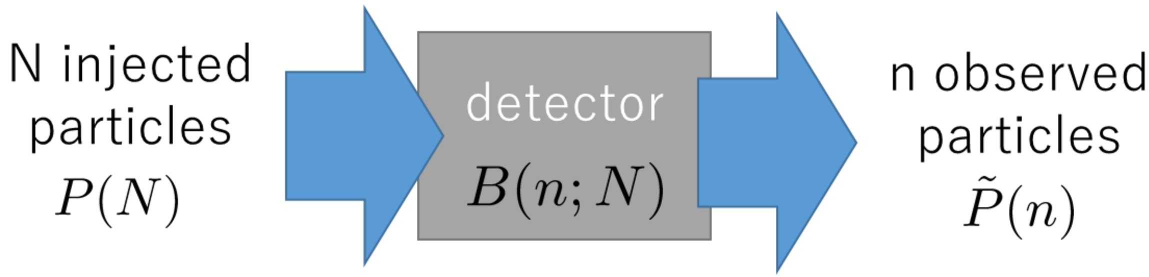
Binomial model:

$$\langle\langle n\rangle\rangle_{\rm c} = \xi_1\langle N\rangle_{\rm c},$$

$$\langle\langle n^2\rangle\rangle_{\rm c} = \xi_2\langle N\rangle_{\rm c} + \xi_1^2\langle N^2\rangle_{\rm c},$$

$$\langle\langle n^3\rangle\rangle_{\rm c} = \xi_3\langle N\rangle_{\rm c} + 3\xi_2\xi_1\langle N^2\rangle_{\rm c} + \xi_1^3\langle N^3\rangle_{\rm c},$$

My Suggestion 2: Understand Your Detector



$$\langle n^m \rangle_{N,\mathrm{c}} = f_m(N)$$

can be obtained by your Monte-Carlo simulator

- Check validity of the binomial model
- When $f_m(N)$ are linear functions
  → use efficiency correction formula in MK2016.
- Non-linearity can be (in part) included by the new formulas.

Some Comments…

- Non-Binomial effects would be estimated perturbatively.
- What is the efficiency? I actually do not understand this concept.