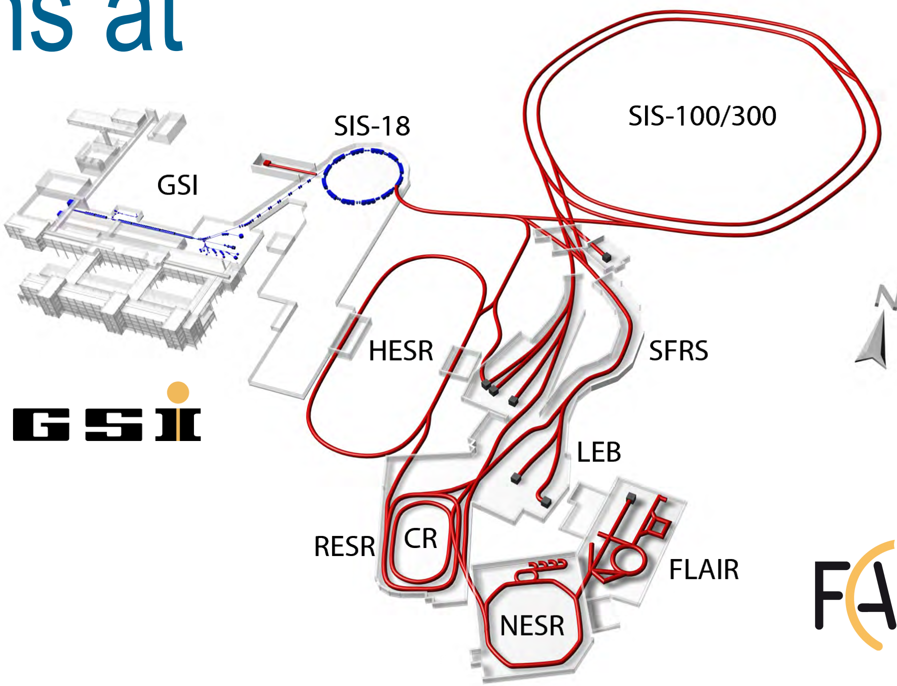
Higher-Order Moments of Proton-Number Fluctuations in Au+Au Collisions at 1.23A GeV with HADES

Melanie Szala



Outline

Motivation

HADES spectrometer

Higher order moments of proton number fluctuations

- Efficiency corrections
- Volume fluctuations
- Results

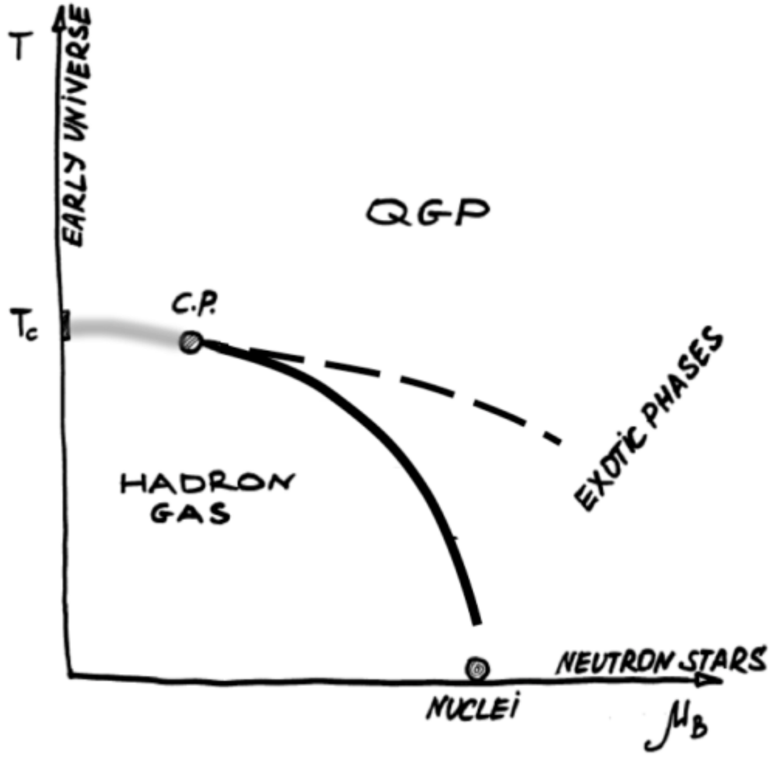
Conclusions

Motivation

- Cross-over at low μ_B and
  1st-order phase transition at high μ_B
  → Critical point

- Event-by-event fluctuations of conserved quantities are expected to be a signature of discontinuity in the QCD phase diagram
  - Charge Q / **baryon number B** / strangeness S

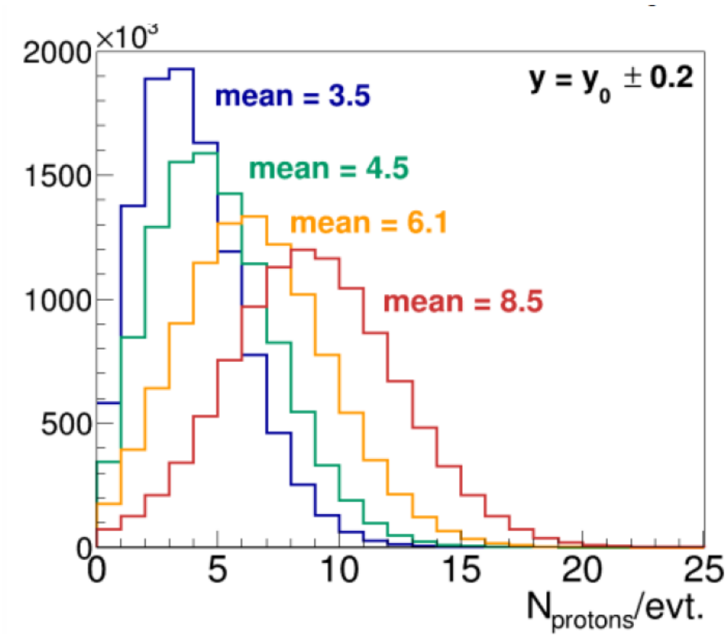
- Experimental observable:
  - Cumulants of event-by-event net-particle multiplicity distributions
  - **(Net-)proton** (proxy for net-baryon)



Experimental Observables

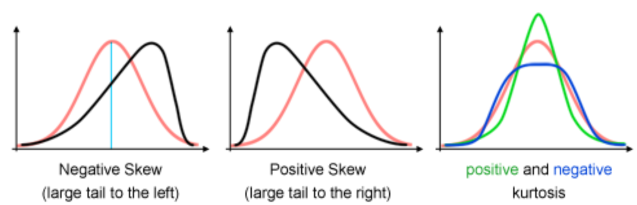
Cumulants of event-by-event net-particle multiplicity distributions

- **(Net-)proton** (proxy for net-baryon)



0-10%, 10-20%, 20-30%, 30-40%

Mean	M	$=$	K_1
Variance	σ^2	$=$	K_2
Skewness	S	$=$	K_3/σ^3
Kurtosis	κ	$=$	K_4/σ^4



Cumulant ratios to cancel volume effects:

$$S\cdot\sigma = \frac{K_3}{K_2} \qquad \kappa\cdot\sigma^2 = \frac{K_4}{K_2}$$

→ Cumulant rations cancel only mean volume effects!

HADES Spectrometer

High **A**cceptance **D**i**E**lectron **S**pectrometer

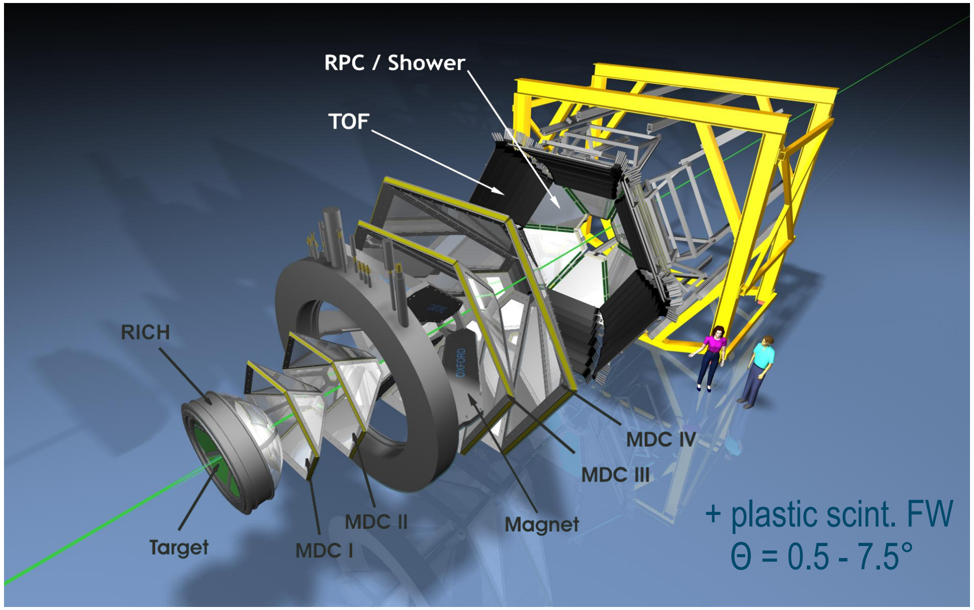
Located at SIS18, GSI

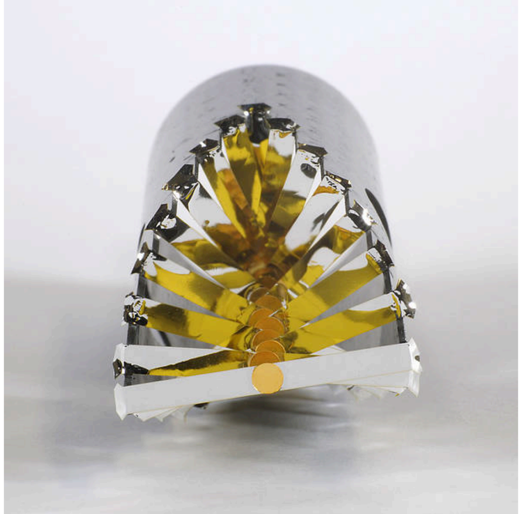
Large acceptance

- Full azimuthal and polar angle coverage of $\Theta = 18 – 85°$

Fast detector

- Trigger rate up to 8kHz (large statistics)

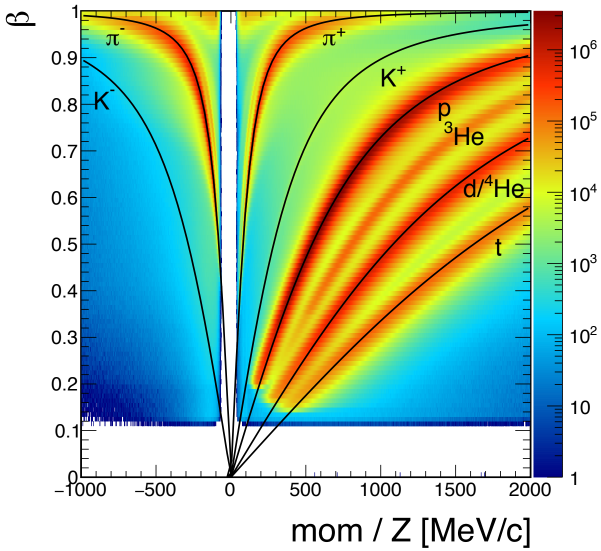


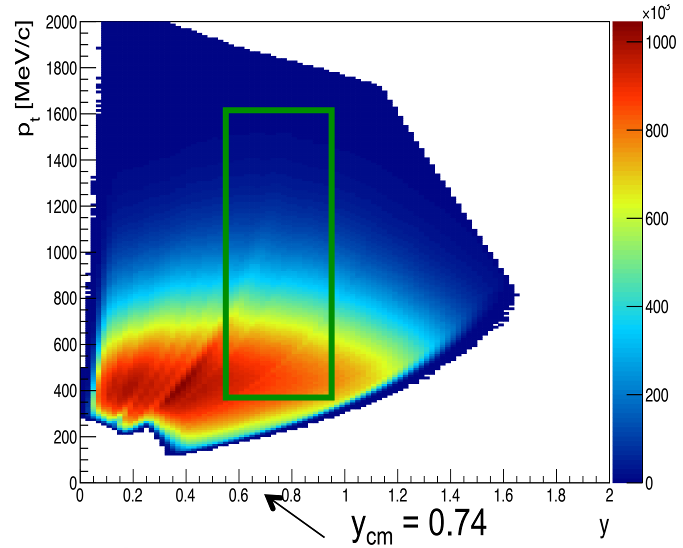


Au + Au @1.23 AGeV, $\sqrt{s_{NN}}$=2.41 GeV

- 15 fold segmented Au target
- 7.4 x 10^9 events recorded
- Trigger on 47% most central collisions

Proton Analysis





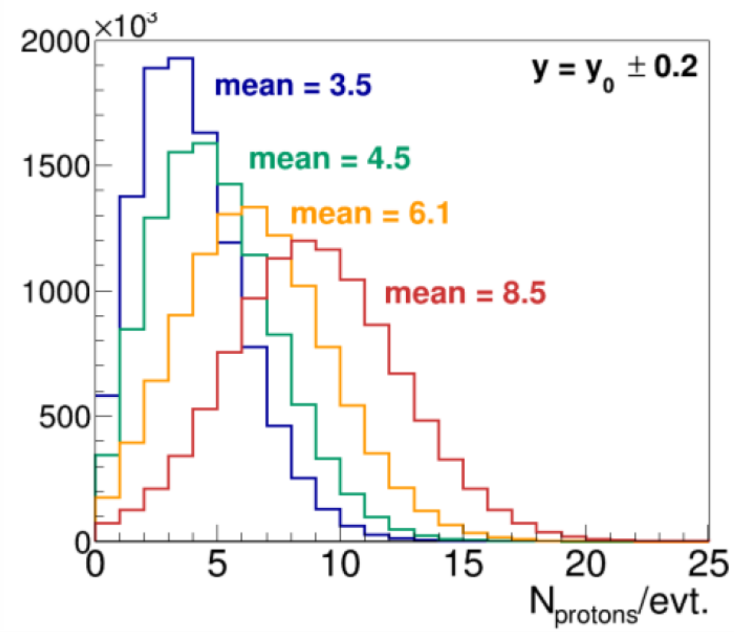
Event selection

- 0-10%, 10-20%, 20-30%, 30-40% central events selected with the Forward Wall

Track selection

- $y = y_{cm} \pm 0.2$
- p_t = 400-1600 MeV/c

If $N_{\bar{p}} = 0$ → Poisson distribution: $S \cdot \sigma = \kappa \cdot \sigma^2 = 1$



Efficiency correction

Efficiency = acc x det. eff x rec. eff

Investigations of efficiency correction in UrQMD simulations:

→ **Correct the moments**

Bzdak & Koch, PRC 86 (2012);

Xiaofeng Luo, arXiv:1410.3914 (2014)

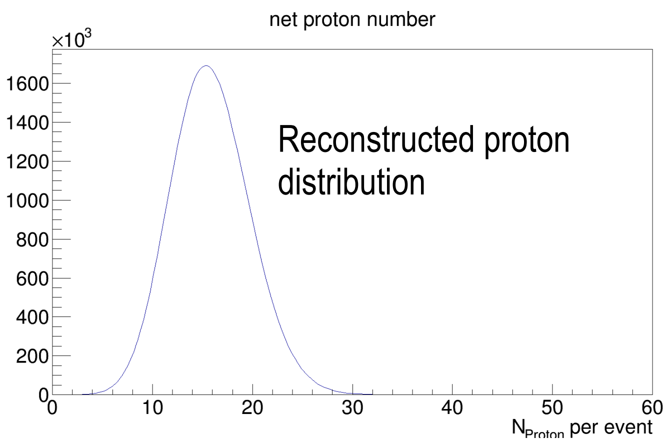
→ **Unfolding**

G. D'Agostino, Nucl. Instr. Meth. A 362 (1995) 487.

J. Albert et al. (MAGIC), Nucl. Instr. Meth. A 583 (2007) 494.

S. Schmitt, J. Instr. 7 (2012) T10003.

Correct the moments



Calculate moments →

$Mean_{measured}$
$Variance_{measured}$
$Skewness_{measured}$
$Kurtosis_{measured}$
$Skew*Sig_{measured}$
$Kurt*Sig^2_{measured}$

Correct moments →

efficiency correction

$Mean_{true}$
$Variance_{true}$
$Skewness_{true}$
$Kurtosis_{true}$
$Skew*Sig_{true}$
$Kurt*Sig^2_{true}$

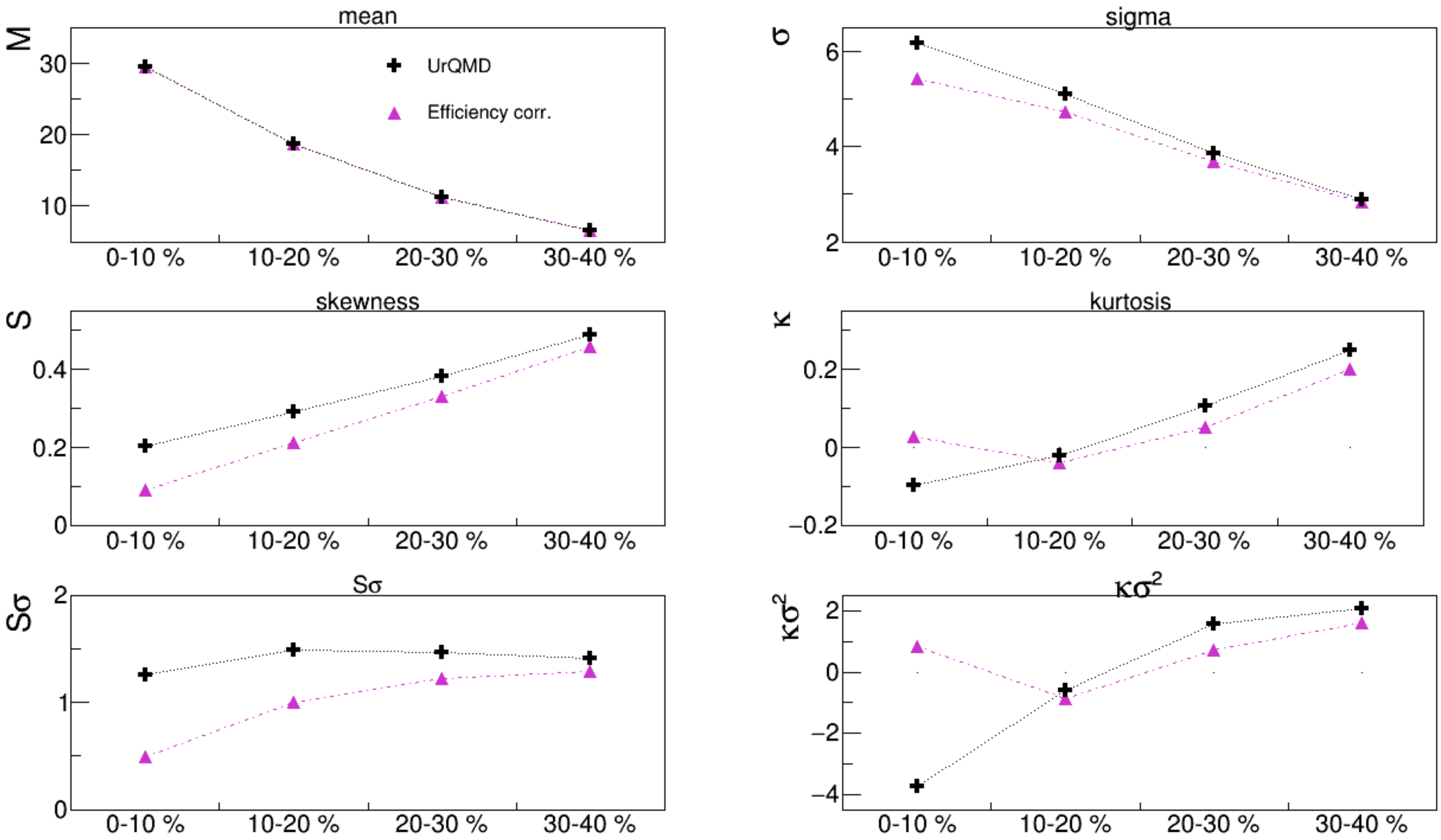
Self-consistency check

- UrQMD protons → reference
- UrQMD + Geant reconstructed protons → efficiency correction

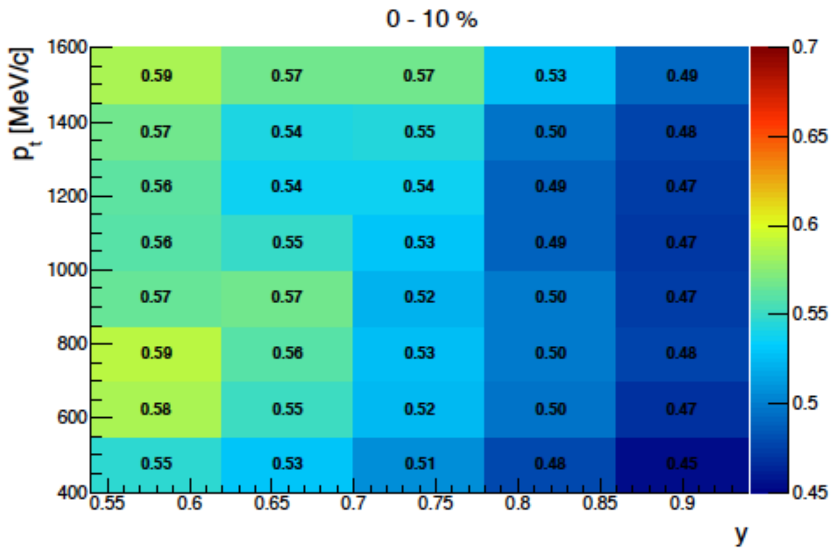
Bzdak & Koch, PRC 86 (2012);
Xiaofeng Luo, arXiv:1410.3914 (2014)

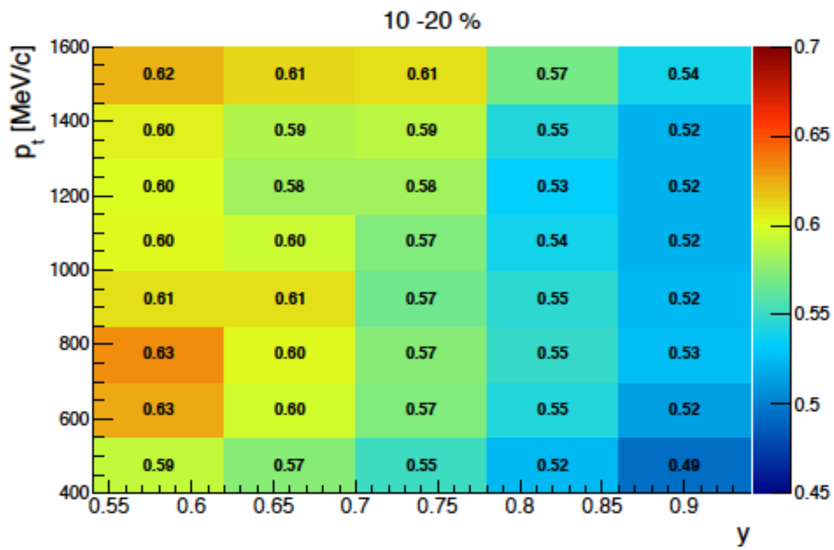
Correct the moments

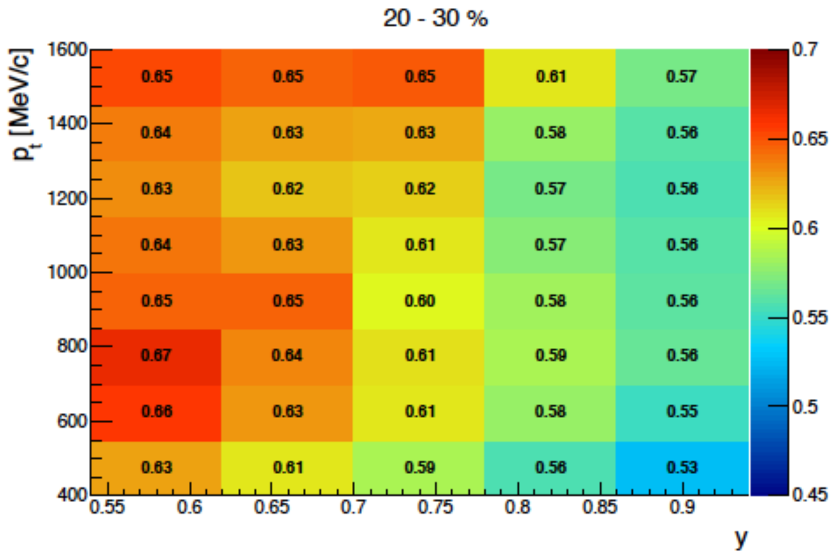
UrQMD

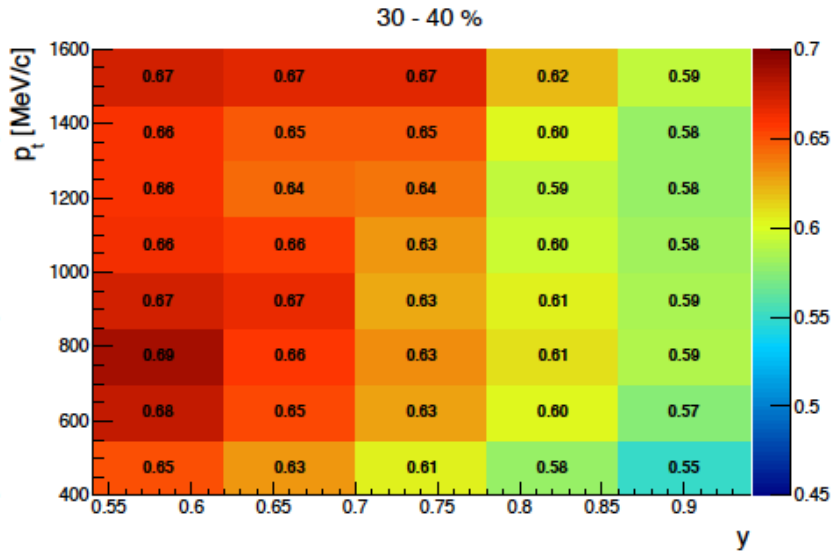


Efficiency correction









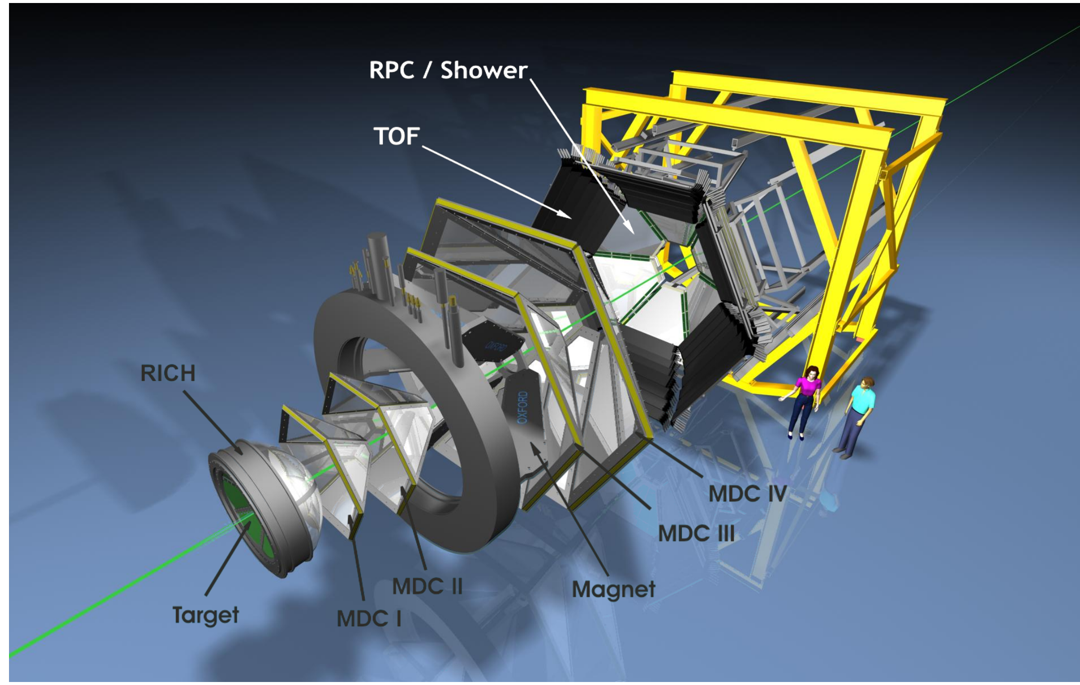
Efficiency correction

Efficiency: strong effect on centrality

Efficiency depends on number of tracks per event

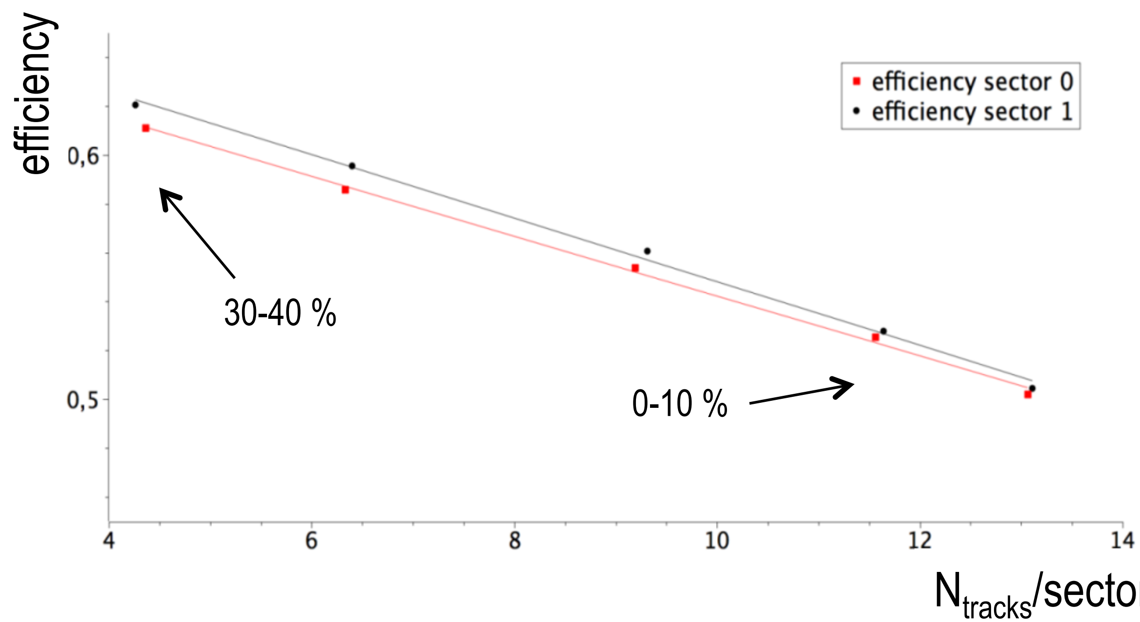
Detector divided into 6 sectors

- Efficiency is sector dependent

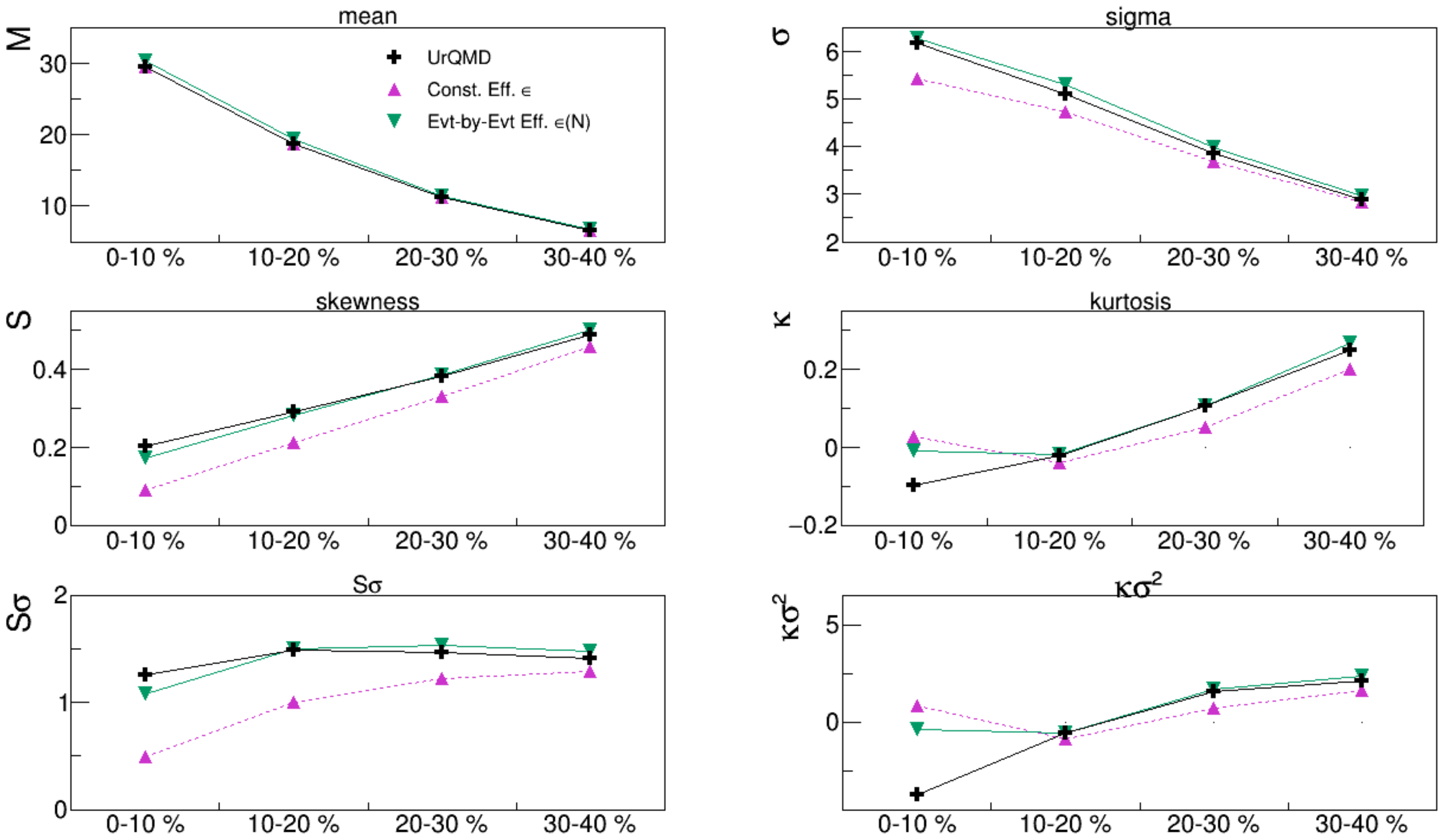


→ Efficiency correction

- Event-by-event
- Particle number in sector
- y and p_t dependent

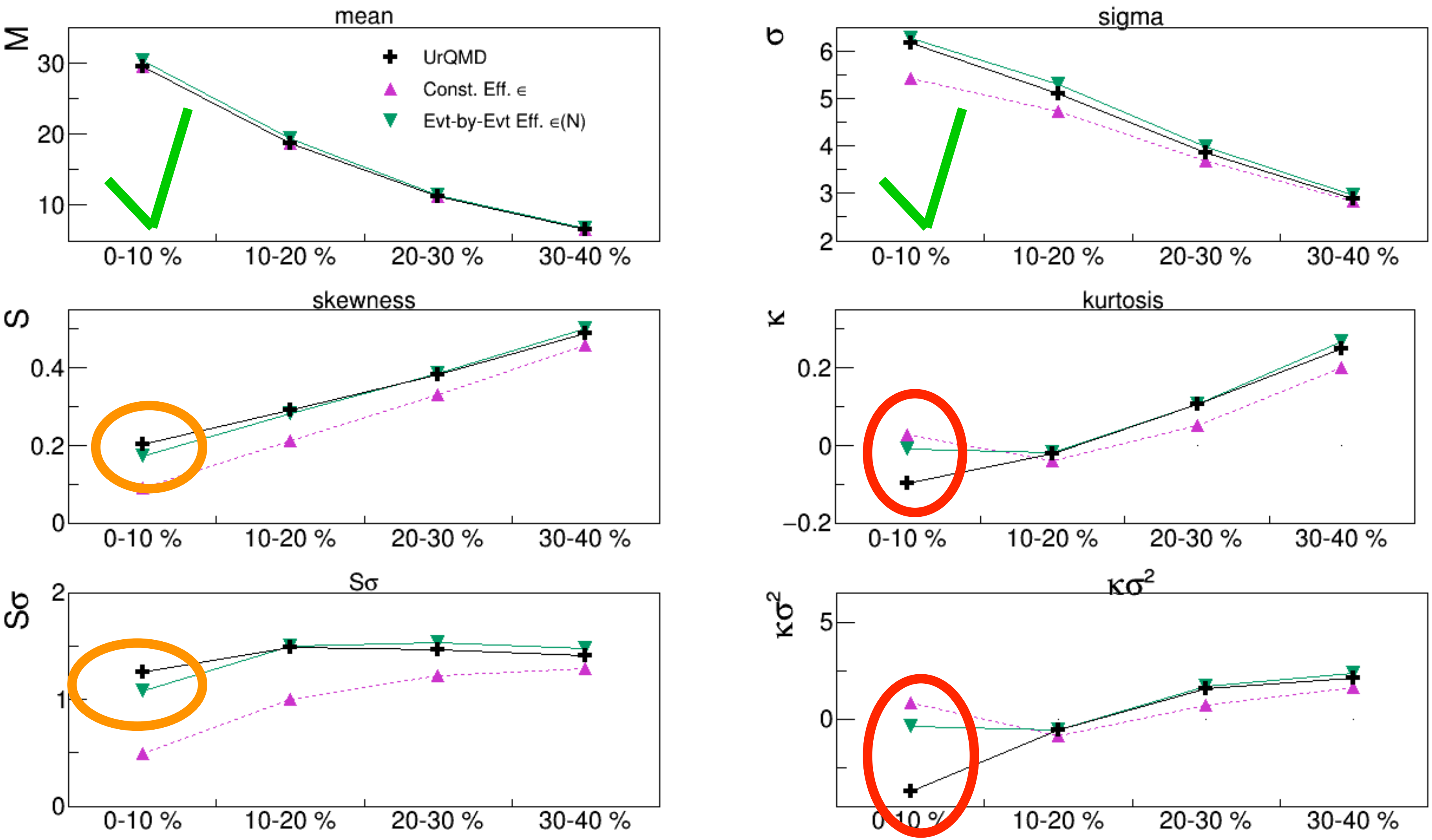


Correct moments – event-by-event -



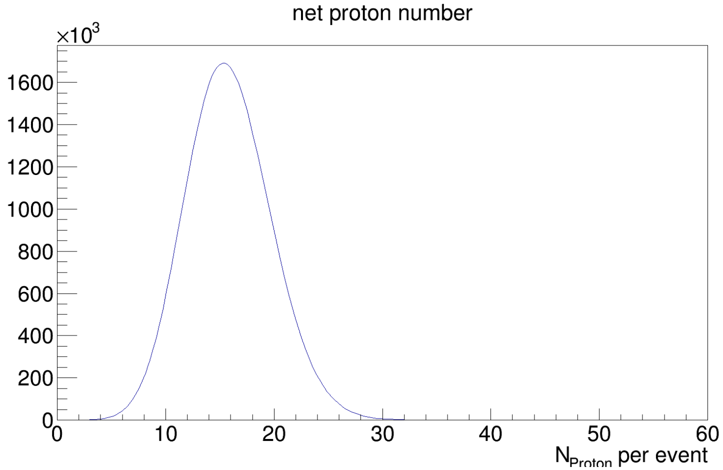
UrQMD

Correct moments – event-by-event -



Unfolding

measured distribution



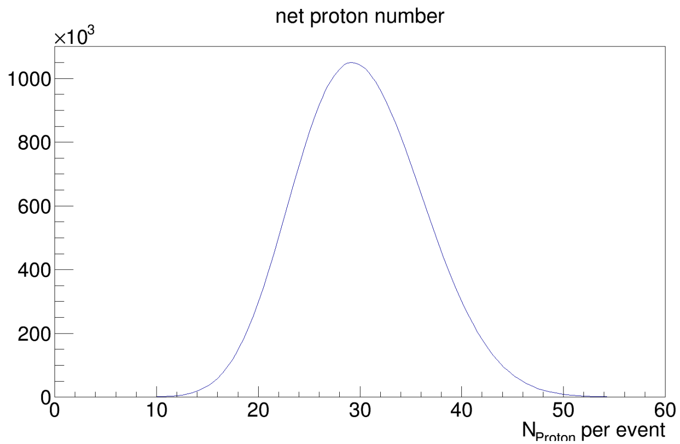
UrQMD + GEANT
(recon. protons)

Unfolding

Matrix inversion

Folding

true distribution



UrQMD protons

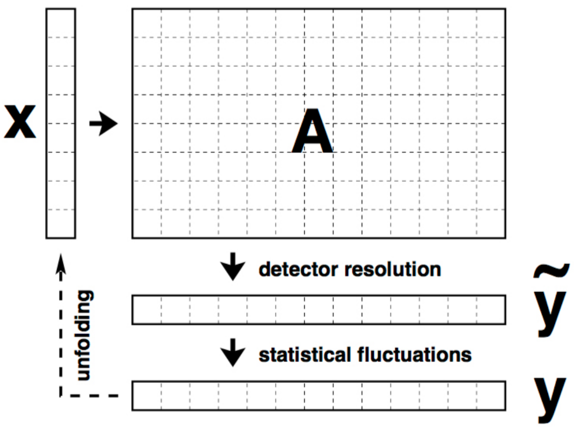
Unfolding procedure

Unfolding via **matrix inversion**

$$\mathbf{y = A \bullet x}$$

→ finding x !

y: measured histogram
x: true histogram
A: response matrix



Unfortunately, **A** is often quasi-singular and can not be inverted (ill-conditioned problem!)

ROOT package by S. Schmitt → **TUnfold, TUnfoldSys, TUnfoldDensity**

Minimize in a least-squares procedure the „Lagrangian“:

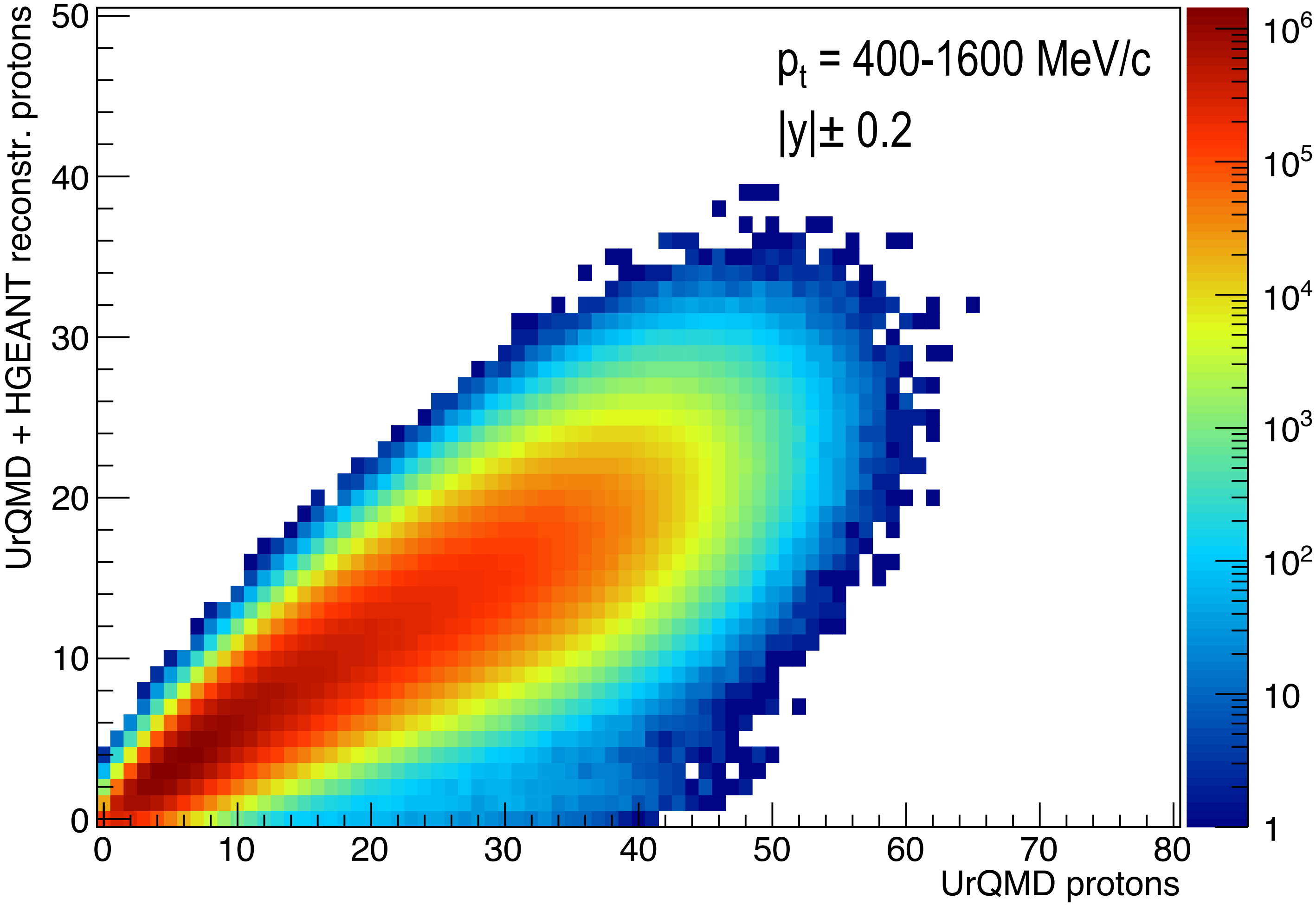
$$\begin{aligned}\mathcal{L}(x,\lambda) =& \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \\ \mathcal{L}_1 =& (\boldsymbol{y} - \mathbf{A}\boldsymbol{x})^{\mathsf{T}} \mathbf{V_{yy}}^{-1} (\boldsymbol{y} - \mathbf{A}\boldsymbol{x}), \\ \mathcal{L}_2 =& \tau^2 (\boldsymbol{x} - f_b \boldsymbol{x_0})^{\mathsf{T}} (\mathbf{L}^{\mathsf{T}}\mathbf{L}) (\boldsymbol{x} - f_b \boldsymbol{x_0}), \\ \mathcal{L}_3 =& \lambda (Y - \boldsymbol{e}^{\mathsf{T}} \boldsymbol{x})\end{aligned}$$

L_1: least square minimization

L_2: describes regularisation

L_3: area constraint

Response function



Regularization

τ is a free parameter controlling the strength of regularization

parameter is typically difficult to choose using only a priori information

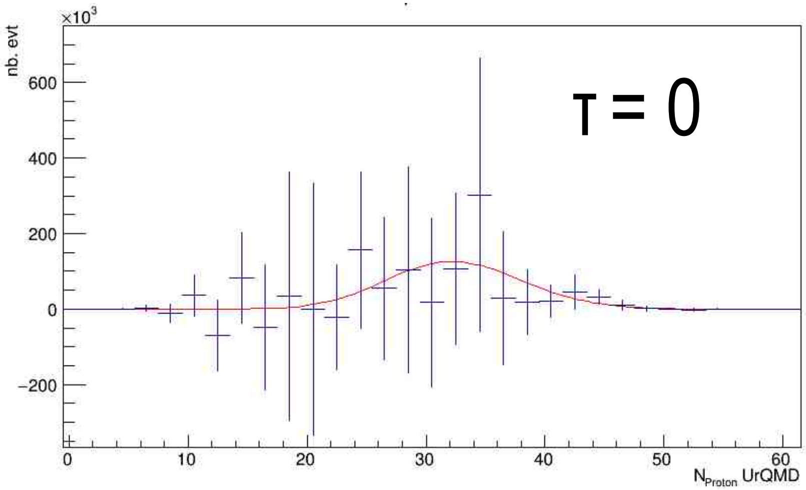
- But its value usually has a major impact on the unfolded spectrum

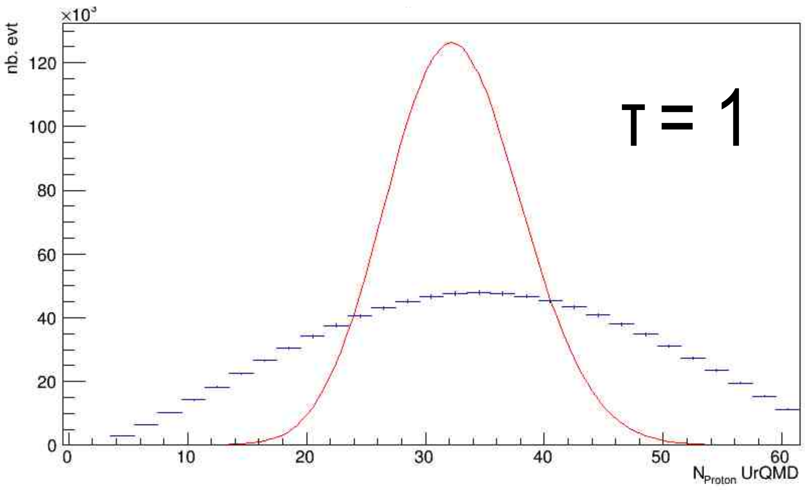
several methods to find τ

in TUnfold implemented methods

- L-curve scan
- Minimising global correlation coefficients

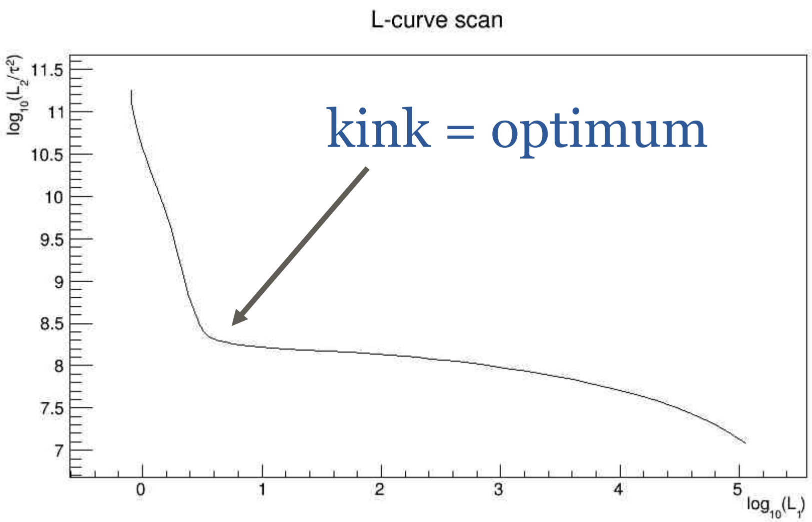
red = original distribution
blue = unfolded distribution



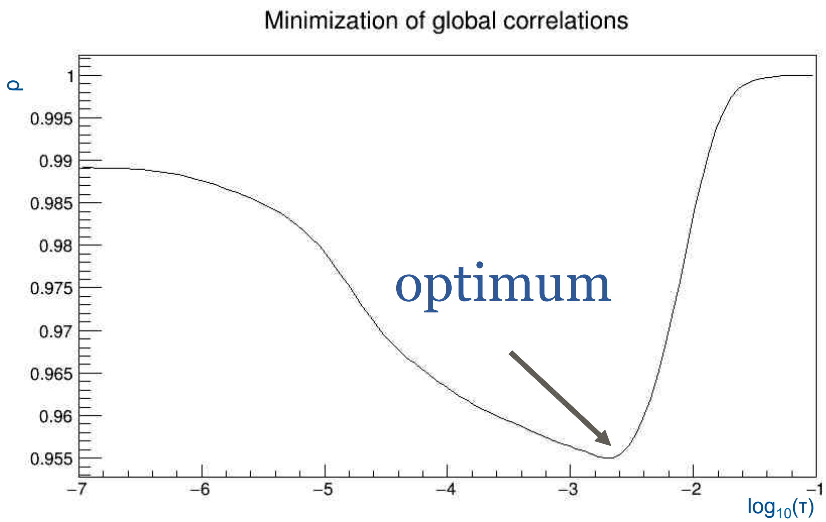


Methods to find best regularization

L-curve Scan

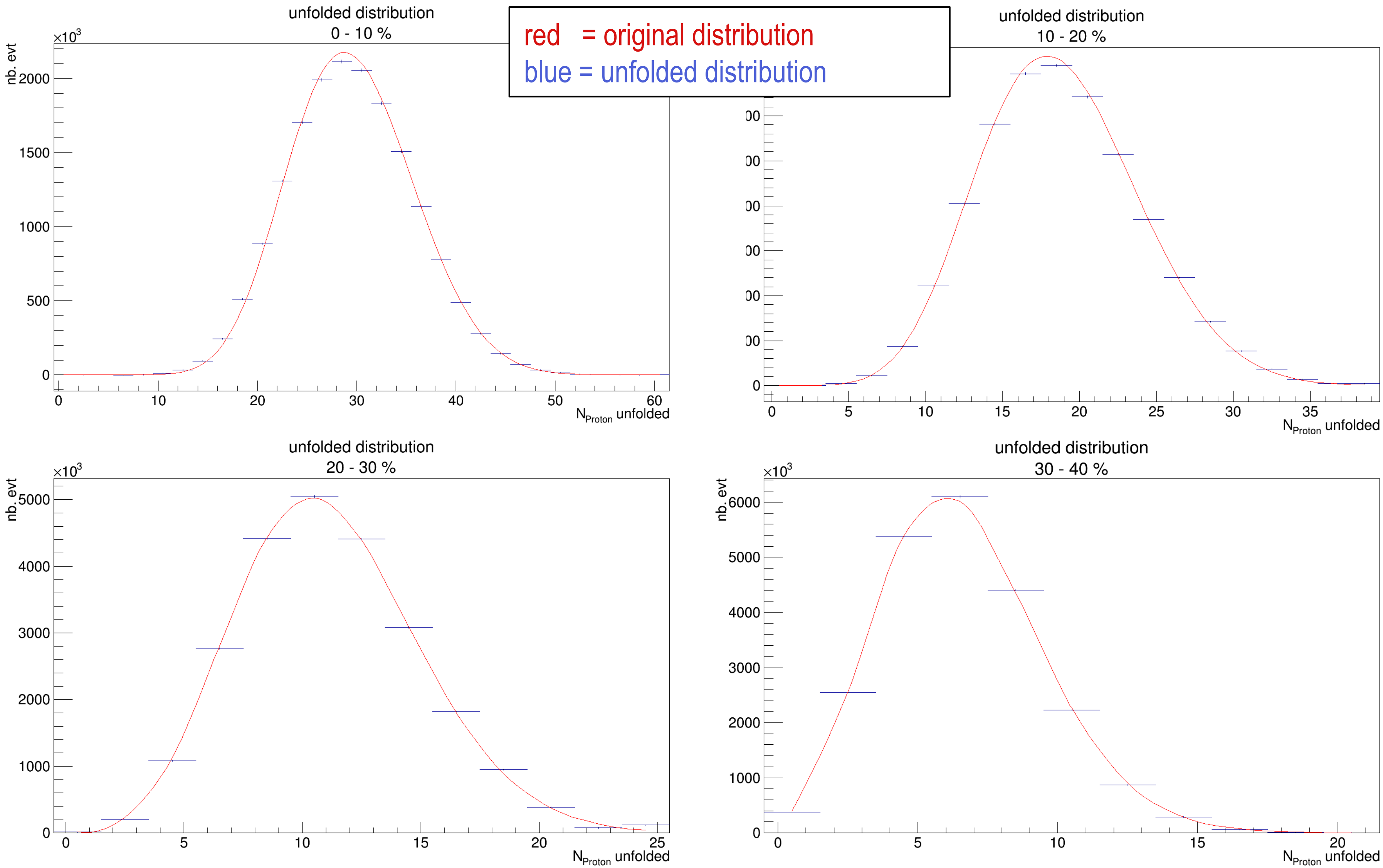


Minimising global correlation coefficients



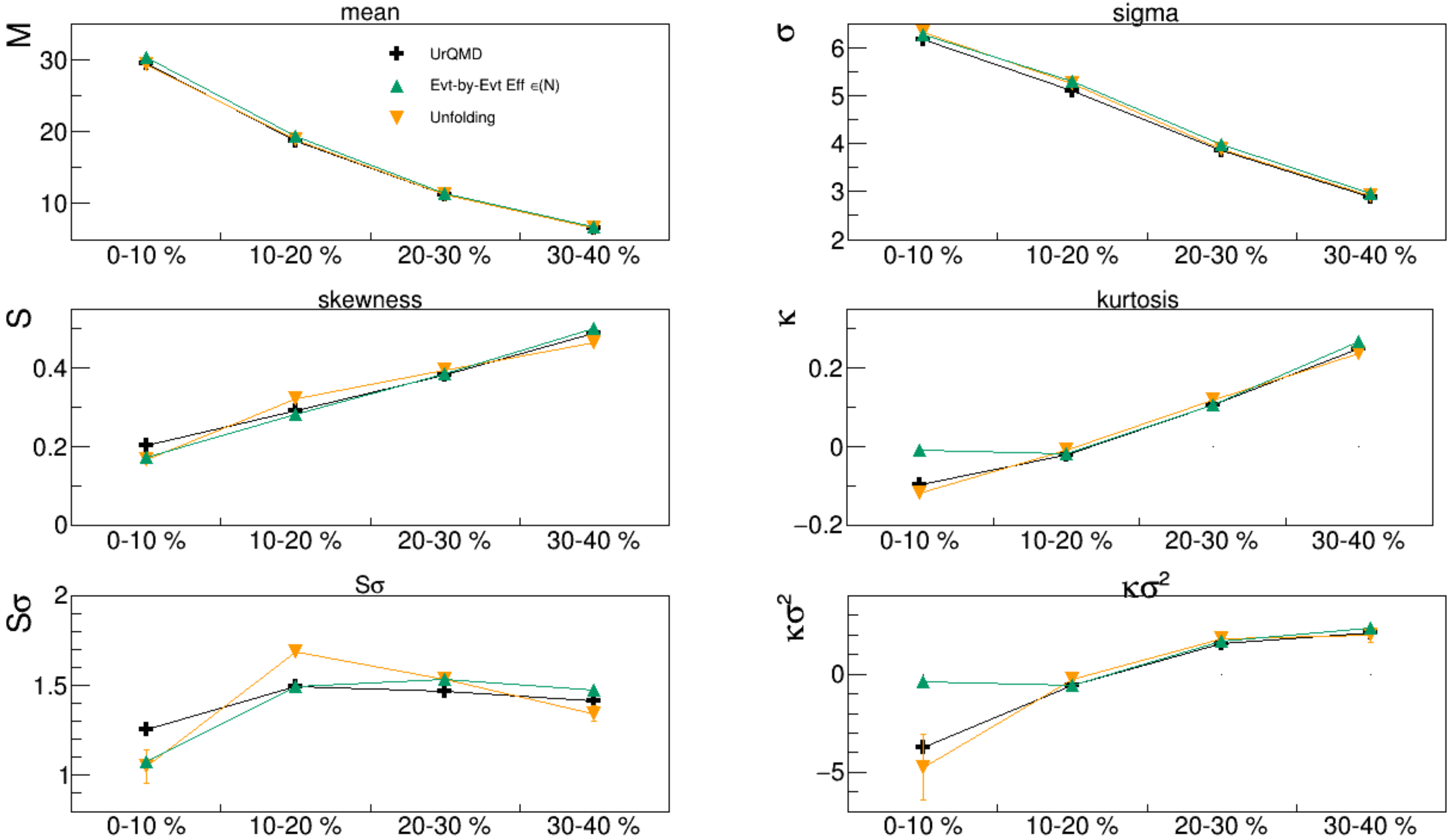
→ $\tau \approx 10^{-4} - 10^{-5}$

Unfolded distributions



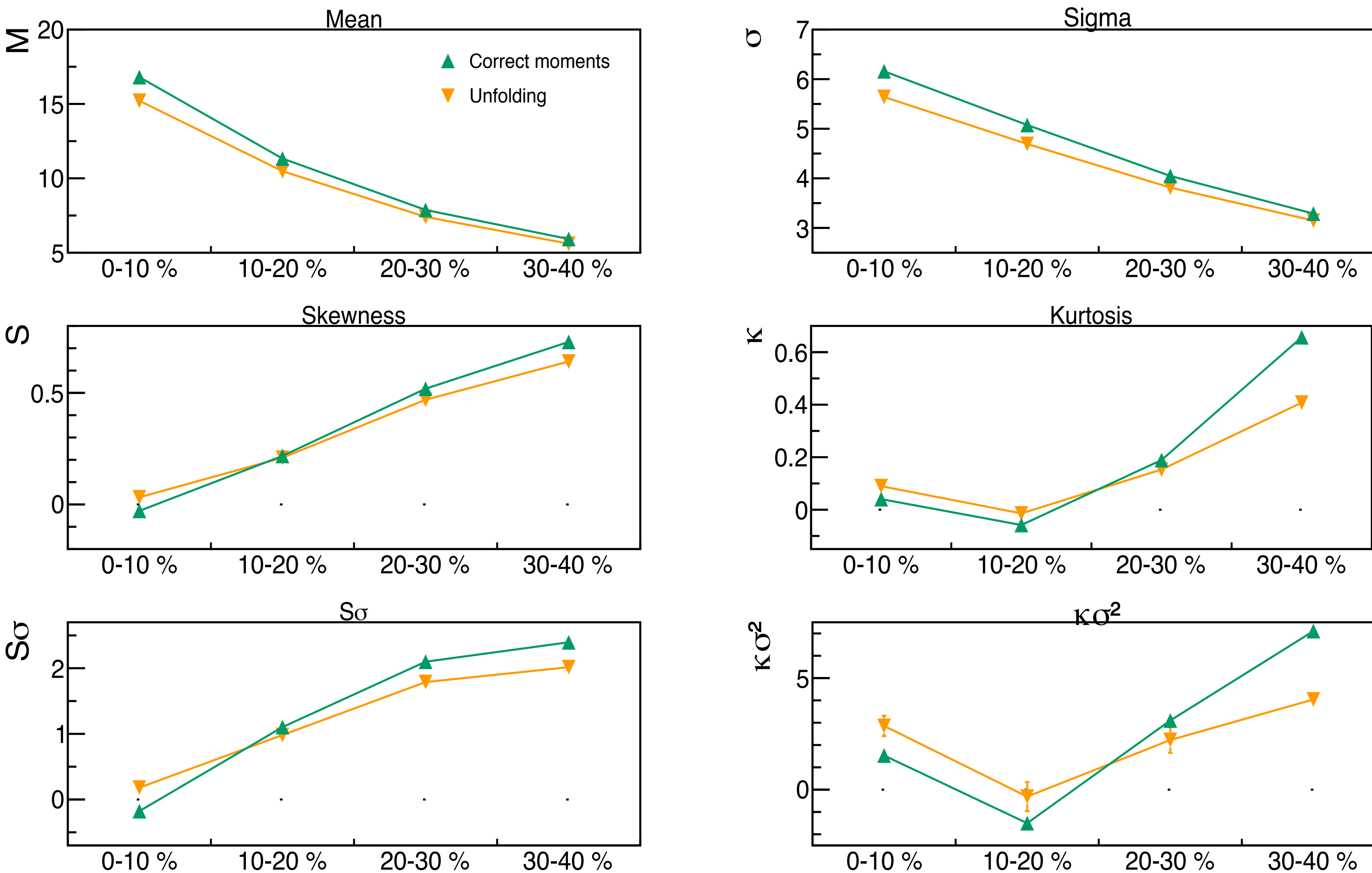
Comparing methods
– Correct moments vs. Unfolding -

UrQMD



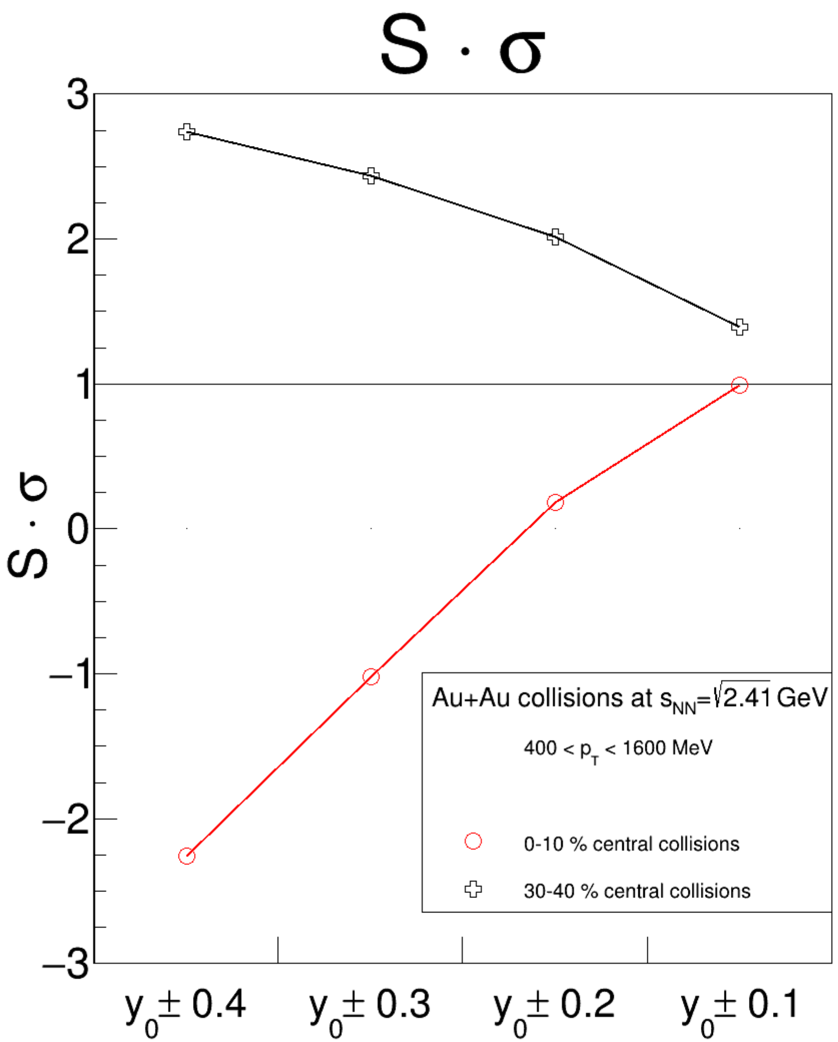
RealData

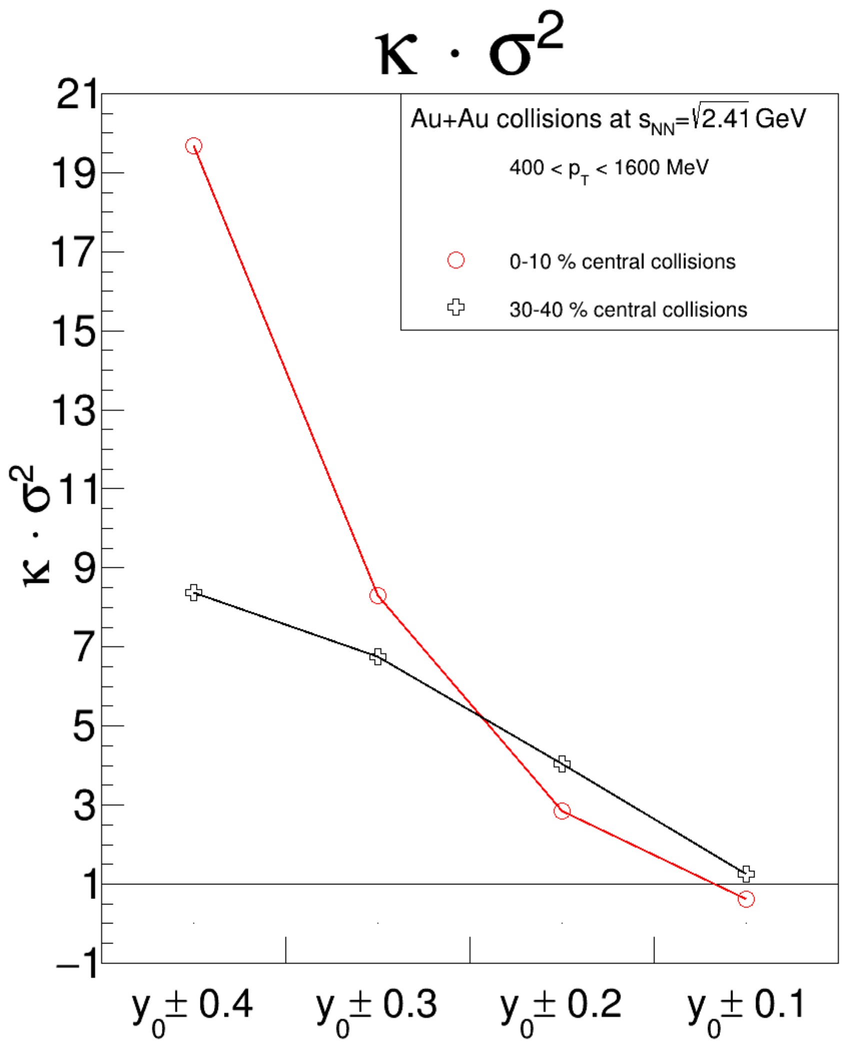
Results (eff. Corr.)



Poissonizer

$\Delta y_{total} \gg \Delta y_{accept} \gg \Delta y_{corr}$



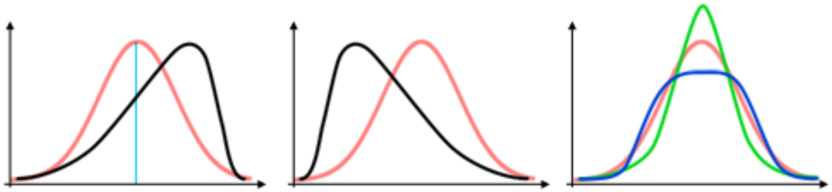


Experimental Observables

Cumulants of event-by-event net-particle multiplicity distributions

- **(Net-)proton** (proxy for net-baryon)

Mean	M	$=$	K_1
Variance	σ^2	$=$	K_2
Skewness	S	$=$	K_3/σ^3
Kurtosis	κ	$=$	K_4/σ^4



Cumulant ratios to cancel volume effects

$$S \cdot \sigma = \frac{K_3}{K_2} \qquad \kappa \cdot \sigma^2 = \frac{K_4}{K_2}$$

→ Cumulant ratios cancel only mean volume effects!

Volume Fluctuations

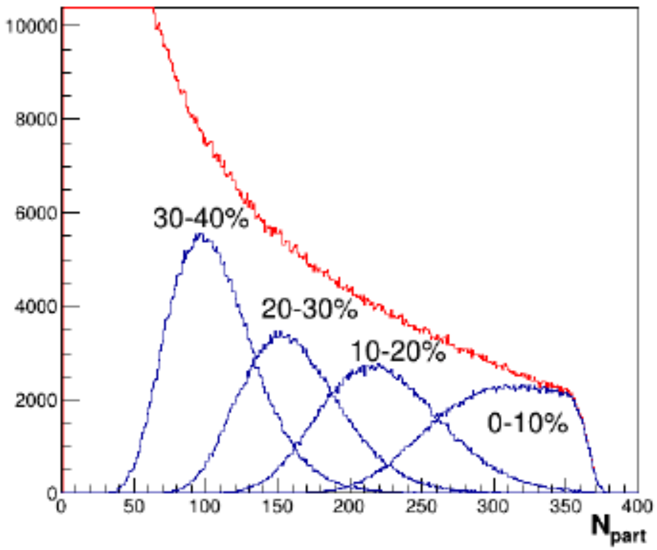
V. Skokov, B. Friman, and K. Redlich, Physical Review C 88, 034911 (2013)

→ The effect of volume fluctuations on cumulants of the net baryon number

$$c_1 = \kappa_1,$$
$$c_2 = \kappa_2 + \kappa_1^2 v_2,$$
$$c_3 = \kappa_3 + 3\kappa_2\kappa_1 v_2 + \kappa_1^3 v_3,$$
$$c_4 = \kappa_4 + \left(4\kappa_3\kappa_1 + 3\kappa_2^2\right)v_2 + 6\kappa_2\kappa_1^2 v_3 + \kappa_1^4 v_4,$$

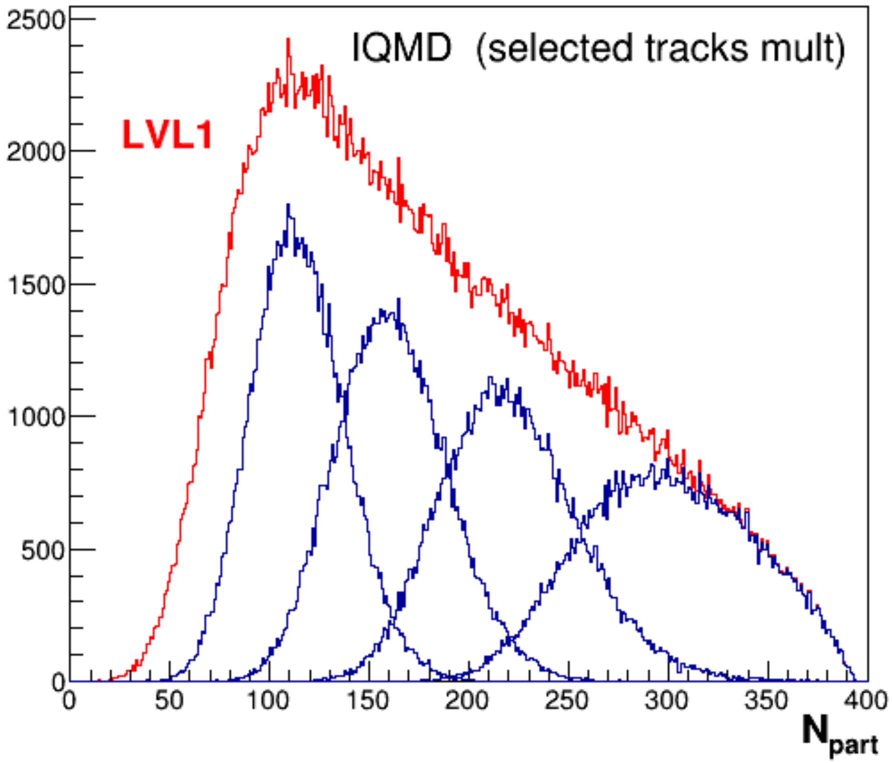
k_n baryon number cumulants
c_n volume affected cumulants
v_n volume fluctuations cumulants

→ Npart ~ volume
→ Modeled (Glauber and iQMD) N_{part} distribution to calculate the volume fluctuation cumulants

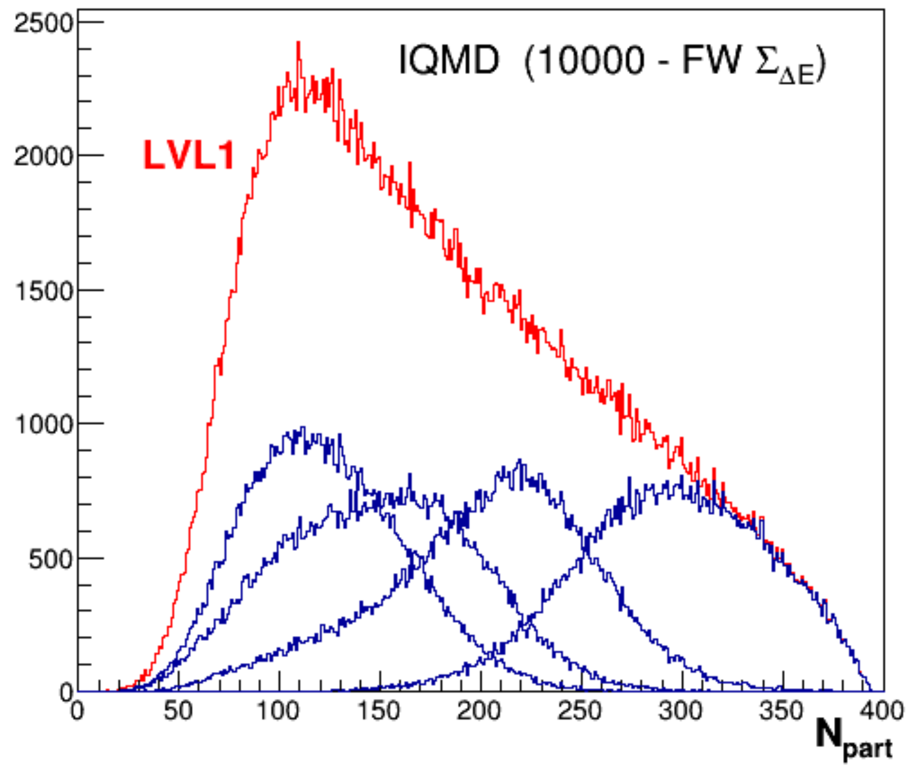


N_{part} from IQMD

Tracks in MDC

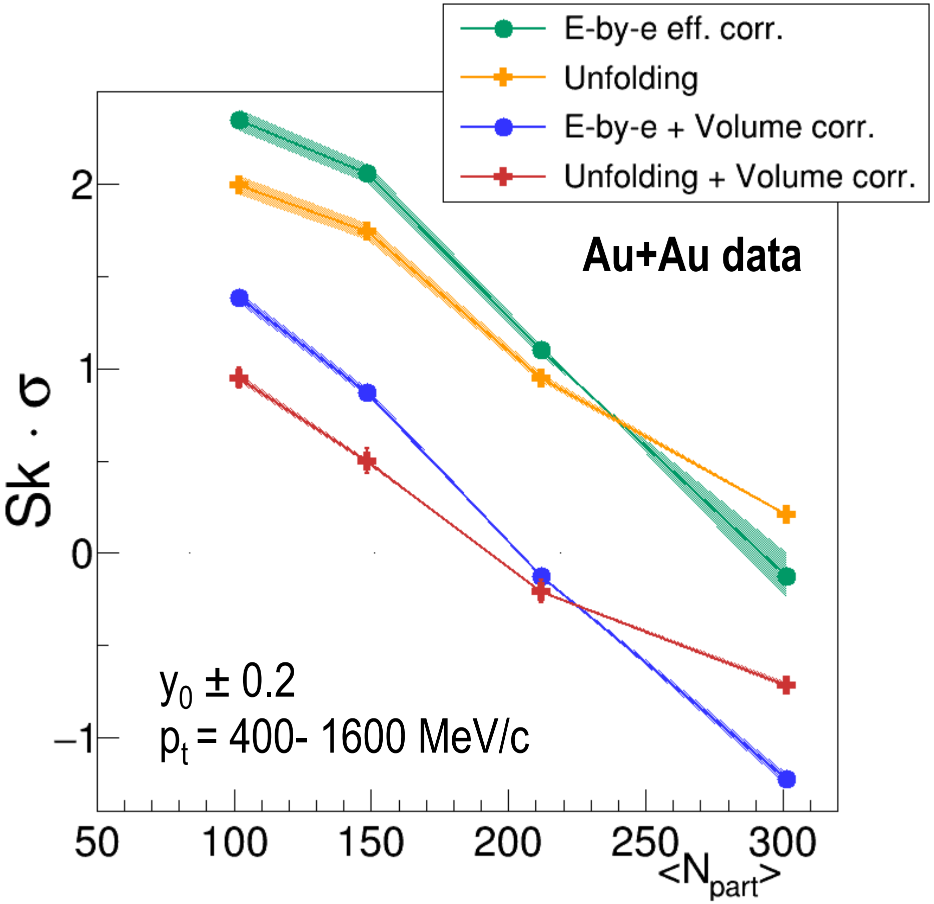


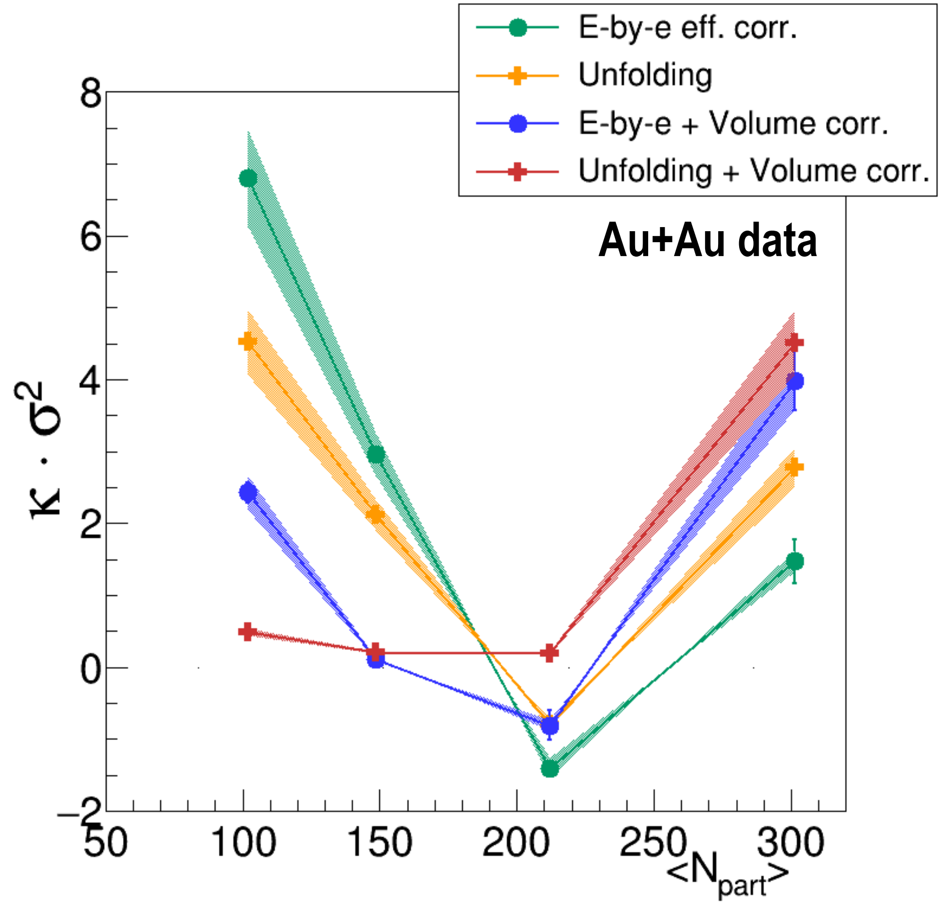
Forward Wall



- For each IQMD event b is translated into N_{part} using Gosset et al. PRC 16
- FW sum of charge has worse resolution

Results with volume corrections (IQMD FW)

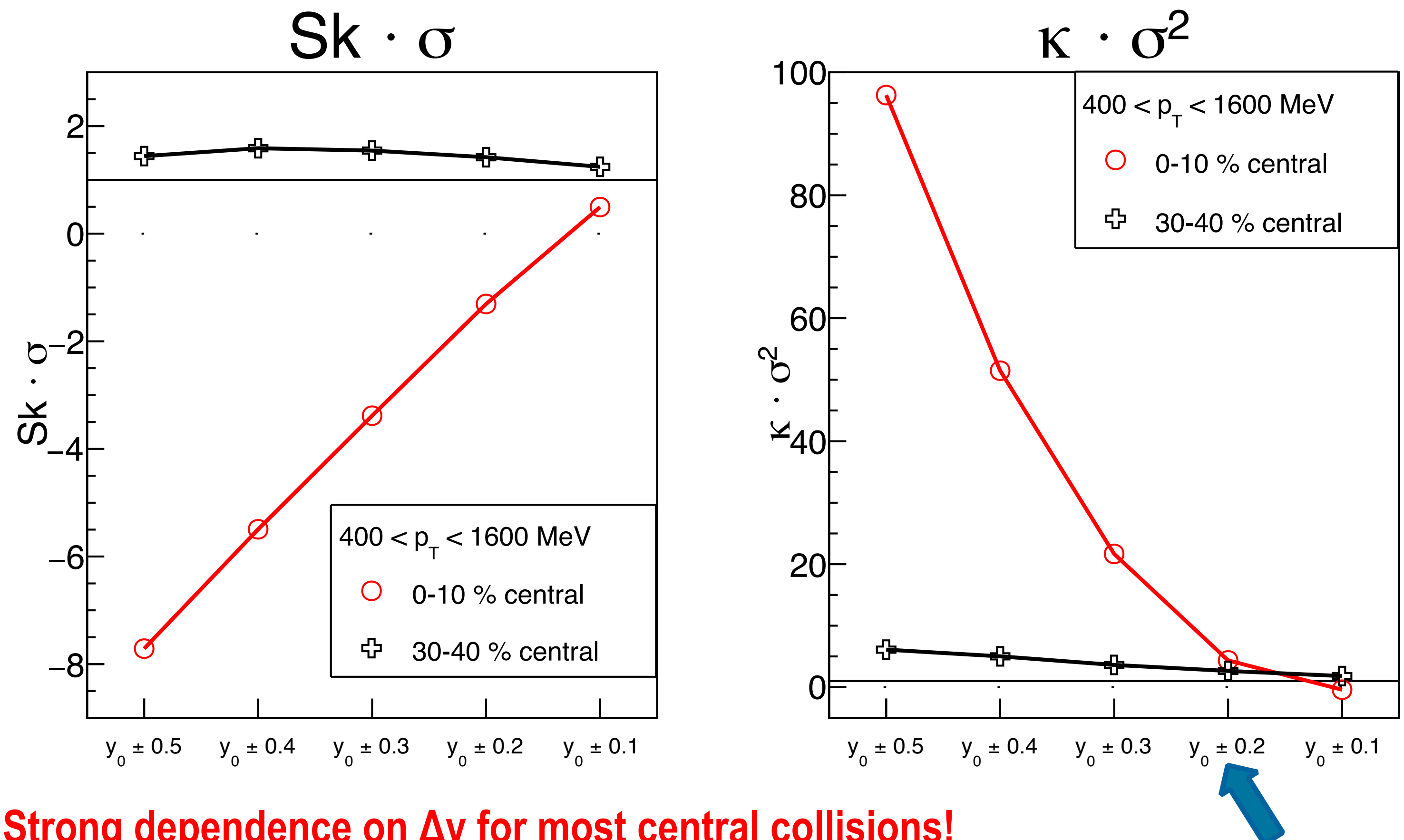




Poisson limit

Poisson limit: $S \cdot \sigma \rightarrow 1$ and $\kappa \cdot \sigma^2 \rightarrow 1$ for $\Delta y \rightarrow 0$

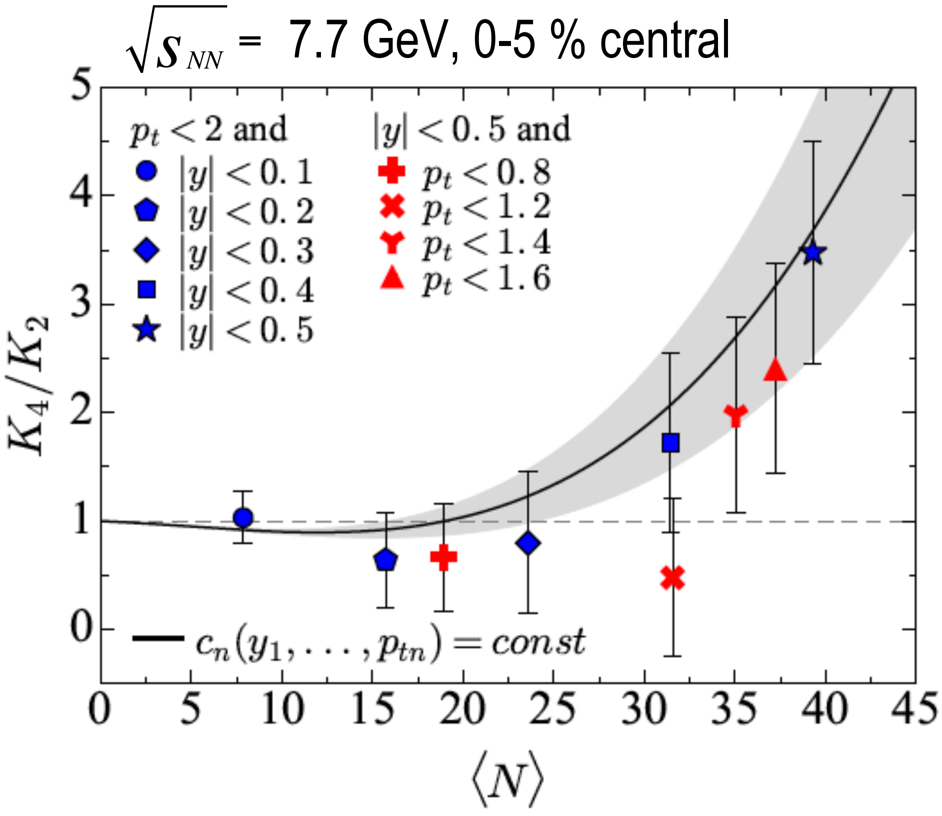
Efficiency corrected protons with volume corrections:

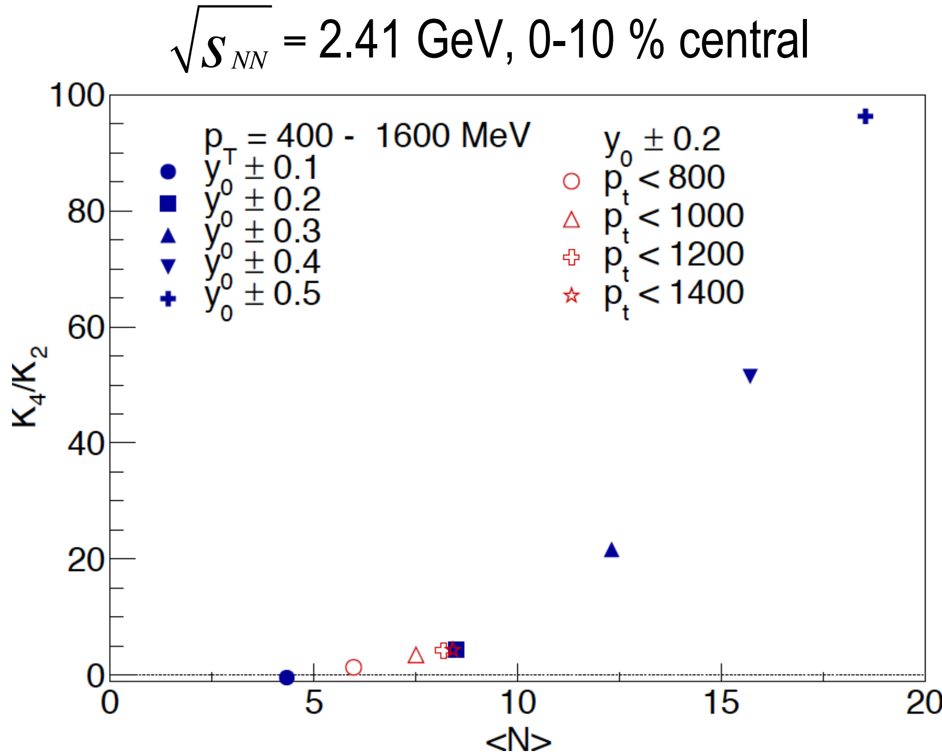


! Strong dependence on Δy for most central collisions!

p_t and y dependance of $\kappa\sigma^2 = K_4/K_2$

Bzdak, Koch arXiv:1707.02640





Cumulants & multi-particle correlators

Bzdak, Koch & Strodthoff, PRC 95, 054906 (2017) based on STAR data (X. Luo et al., CPOD2014)

- Cumulants mix correlations of different order,e.g. K_4 contains of two-, three- and four-particle correlations

- Measuring couplings of the multi-particle correlation functions could provide cleaner information on possible non-trivial dynamics in heavy-ion collisions
  → extract true correlation functions from the measured cumulants

$$\begin{aligned} K_2 &= \langle N \rangle + C_2, \\ K_3 &= \langle N \rangle + 3C_2 + C_3, \\ K_4 &= \langle N \rangle + 7C_2 + 6C_3 + C_4, \end{aligned}$$

and

$$\begin{aligned} C_2 &= -\langle N \rangle + K_2, \\ C_3 &= 2\langle N \rangle - 3K_2 + K_3, \\ C_4 &= -6\langle N \rangle + 11K_2 - 6K_3 + K_4. \end{aligned}$$

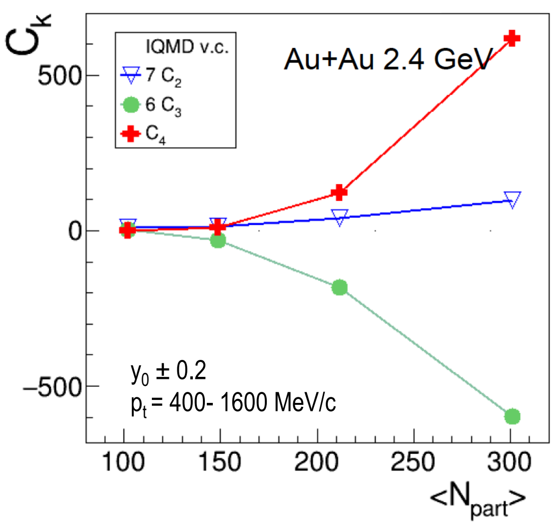
- Reduced correlation functions

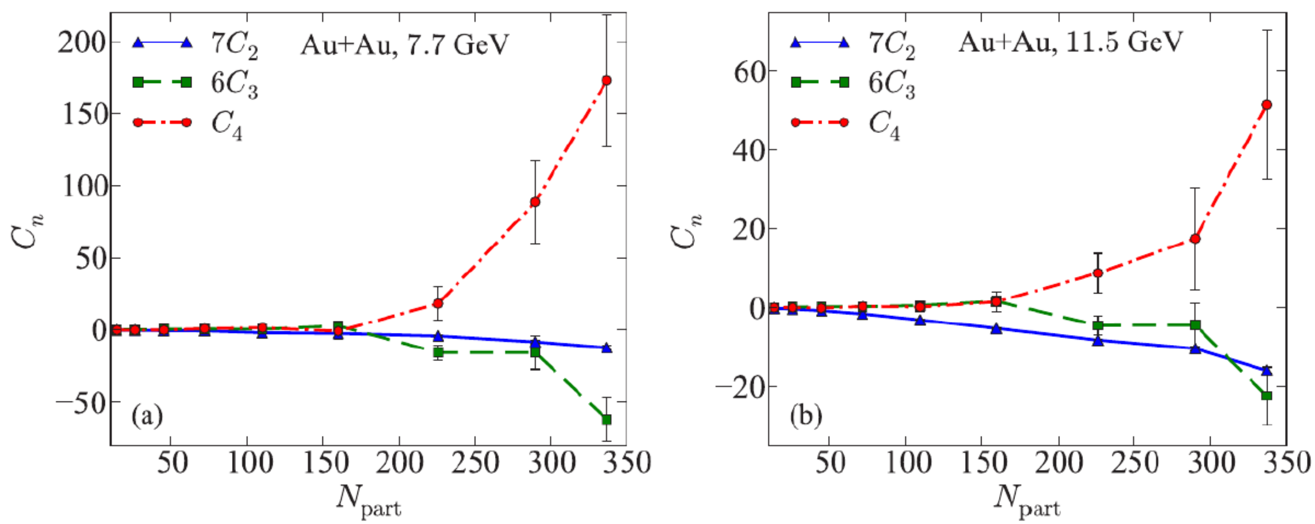
$$C_k = \langle N \rangle^k c_k,$$

N_{part} dependance of proton correlations

- C_k vs. N_{part} as a better approach to isolate critical fluctuations:
- Factors to reflect their relative contribution to the fourth order cumulant

$$\kappa_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$





κ_k: cumulants

C_k: multi-particle correlators

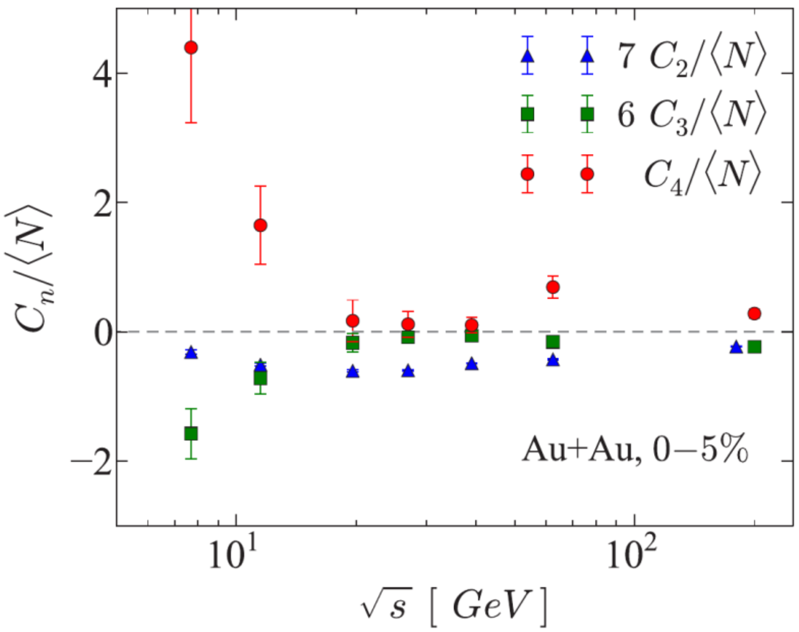
Proton correlations C_n/N_p vs. Beam Energy

HADES: $\sqrt{s}$ = 2.4 GeV

$C_4/N \simeq 36$

$7\,C_2/N \simeq 6$

$6\,C_3/N \simeq -35$



- At energies larger that 19.6 GeV anti-protons become non-negligible, and thus the physical interpretation is less clear.
- K4/K2 is dominated by C_4 at low energies and by C_2 at 19.6 GeV .

→ should not forget that there are sources of correlations other that critical dynamics. need to be removed and understood

What we did...

1) Systematic checks in simulations (UrQMD + Geant)
    - Centrality selection
    - Efficiency correction
        - (Pt-y dependend eff correction)
        - Pt-y and track multiplicity dependend eff correction
        - Unfolding

2) Au+Au data
    - Centrality with EvtCharaClass: FWSumCharge
    - Two different eff correction methods

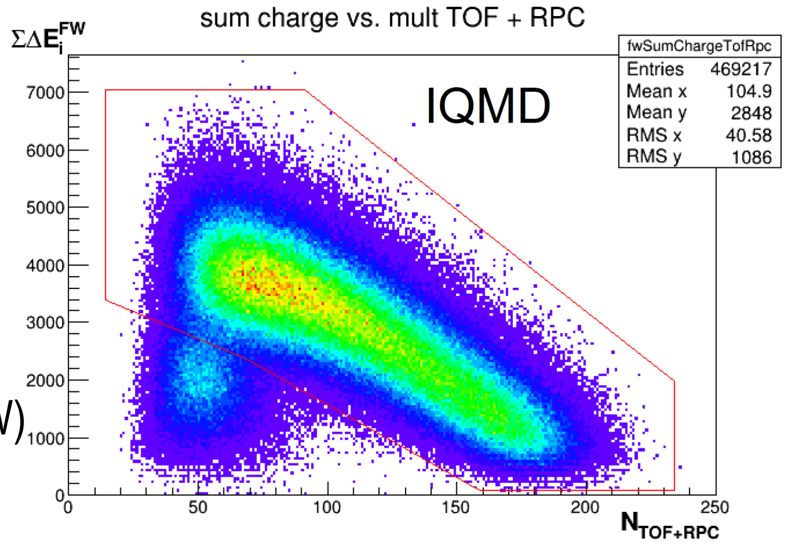
    → Higher order moments of proton distribution

    → Higher order moments of proton and deuteron distributions

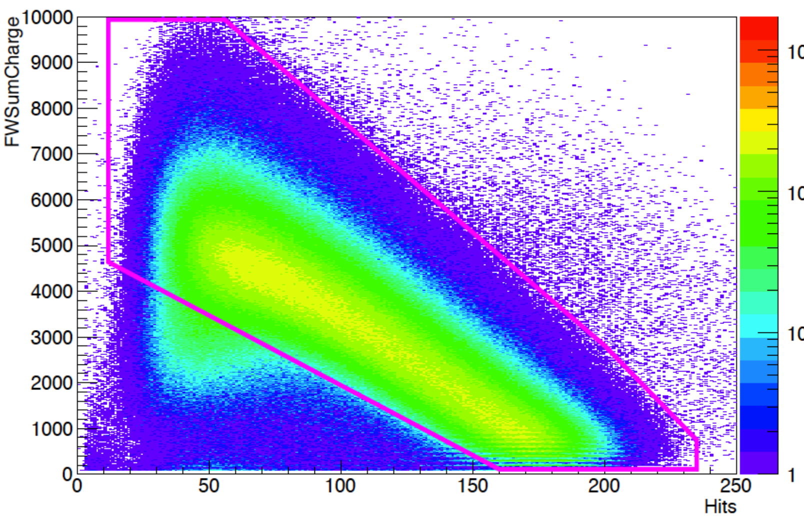
3) Volume fluctuations
    - Glauber model
    - IQMD

What to do...

1) Systematic checks in simulations (IQMD + Geant)
   - Centrality selection
     - FW
     - Resolution vs autocorrelation (Hits/Tracks/FW)
   - Include fragments (efficiency, ...)



2) Possible contamination of moments
   - Resolution centrality estimation
   - Au+C reactions
   - Pile up effects
   - ...



Conclusions

Study of centrality selection and efficiency correction in simulation

Introduction of two different eff. correction methods

- Within error bars, both methods deliver compatible results

First look to volume fluctuation correction

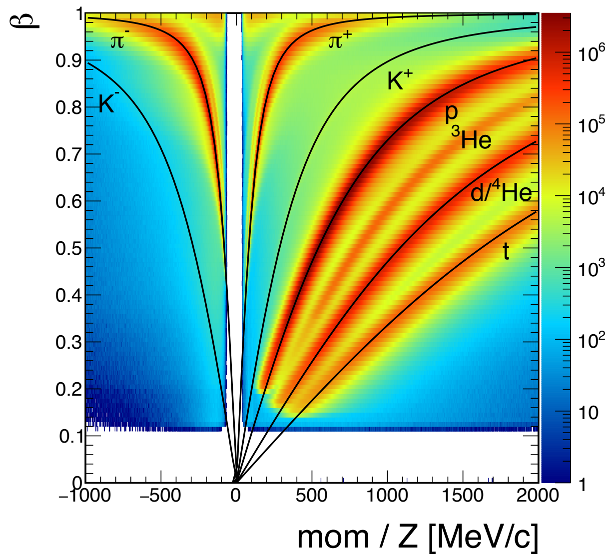
- Volume correction influences higher order moments

Choice of phase-space bin ? (p_t, y)

Fluctuations on baryon number

- Bound protons?!

Possible contamination of moments



The HADES Collaboration

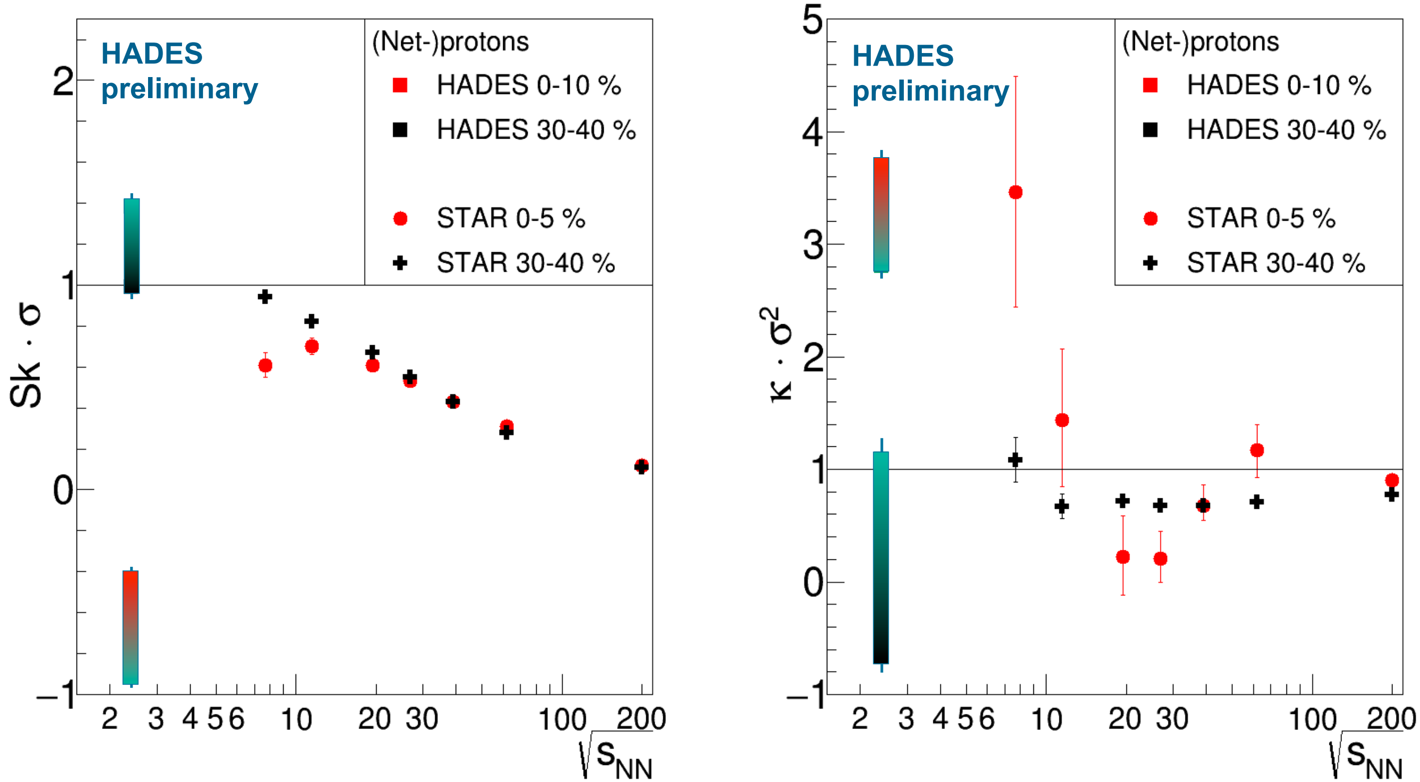
Backup

Comparison with STAR - (Net-) Proton

STAR analysis: Xiaofeng Luo et al., PoS (CPOD2014) 019
arXiv:1503.02558v2

HADES:
p_t = 400 – 1600 MeV/c
$y = y_0$ +- 0.2
Forward Wall
Volume Corrected

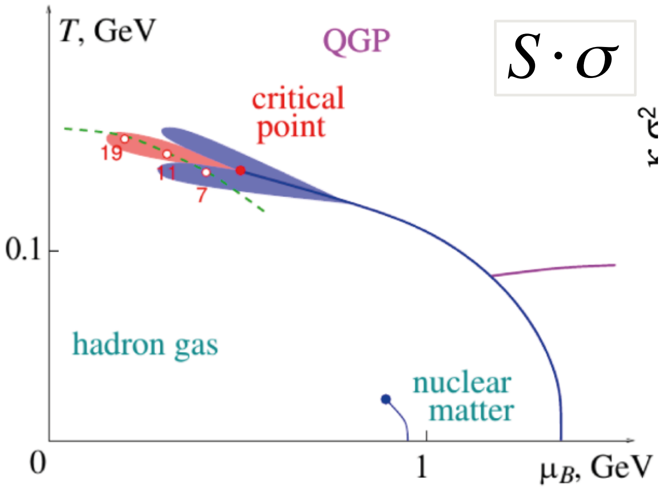
STAR:
p_t = 400 – 2000 MeV/c
$y = y_0$ +- 0.5
NOT Volume Corrected

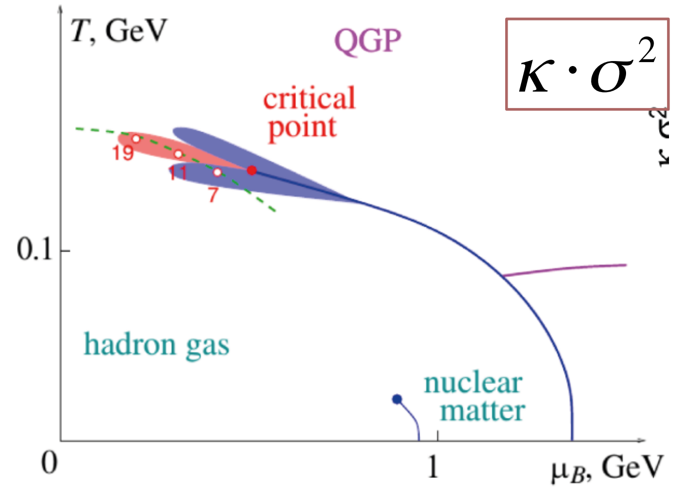


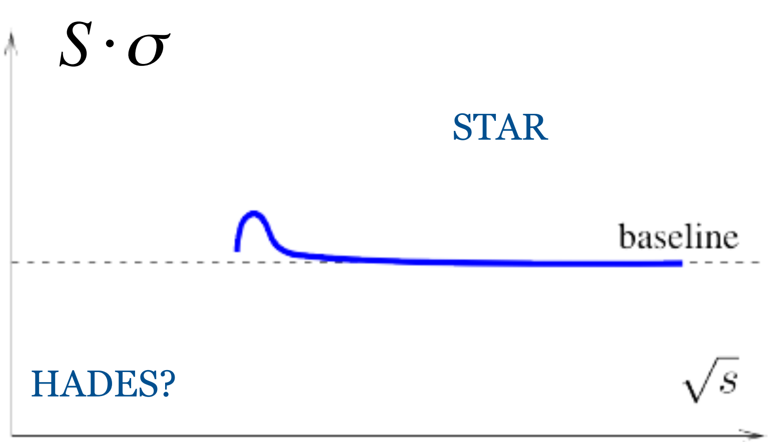
- red/black = unfolding (preferred method) + vol. flucs. corr.
- green = evt-by-evt eff correction of factorial moments + vol. flucs. corr.

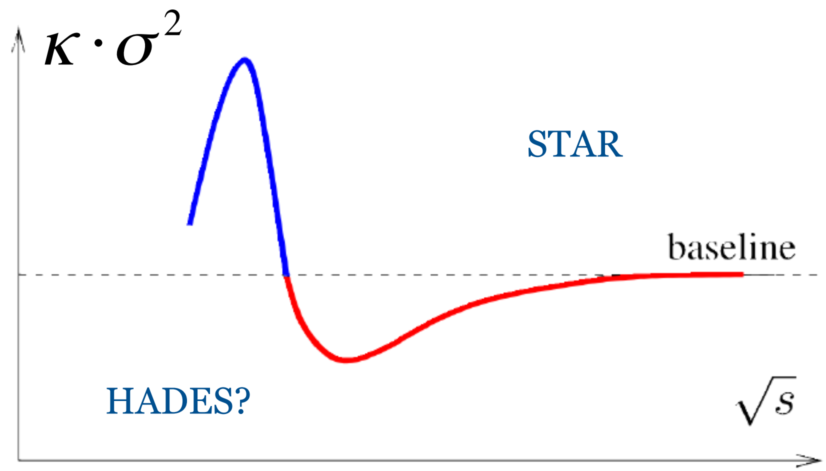
Theoretical Expectation

Misha Stephanov, CPOD 2014









Correct the moments

(1)

$$F_{i,k}(N_p, N_{\bar{p}}) = \left\langle \frac{N_p!}{(N_p - i)!} \frac{N_{\bar{p}}!}{(N_{\bar{p}} - k)!} \right\rangle = \sum_{N_p=i}^{\infty} \sum_{N_{\bar{p}}=k}^{\infty} P(N_p, N_{\bar{p}}) \frac{N_p!}{(N_p - i)!} \frac{N_{\bar{p}}!}{(N_{\bar{p}} - k)!}$$

$$f_{i,k}(n_p, n_{\bar{p}}) = \left\langle \frac{n_p!}{(n_p - i)!} \frac{n_{\bar{p}}!}{(n_{\bar{p}} - k)!} \right\rangle = \sum_{n_p=i}^{\infty} \sum_{n_{\bar{p}}=k}^{\infty} p(n_p, n_{\bar{p}}) \frac{n_p!}{(n_p - i)!} \frac{n_{\bar{p}}!}{(n_{\bar{p}} - k)!}$$

$$F_{i,k}(N_p, N_{\bar{p}}) = \frac{f_{i,k}(n_p, n_{\bar{p}})}{(\varepsilon_p)^i (\varepsilon_{\bar{p}})^k}$$

(2)

$$A_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) = \langle N(x_1)[N(x_2) - \delta_{x_1,x_2}] \ldots [N(x_i) - \delta_{x_1,x_i} - \ldots - \delta_{x_{i-1},x_i}]$$
$$\bar{N}(\bar{x}_1)[\bar{N}(\bar{x}_2) - \delta_{\bar{x}_1,\bar{x}_2}] \ldots [\bar{N}(\bar{x}_k) - \delta_{\bar{x}_1,\bar{x}_k} - \ldots - \delta_{\bar{x}_{k-1},\bar{x}_k}]\rangle$$

$$a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k) = \langle n(x_1)[n(x_2) - \delta_{x_1,x_2}] \ldots [n(x_i) - \delta_{x_1,x_i} - \ldots - \delta_{x_{i-1},x_i}]$$
$$\bar{n}(\bar{x}_1)[\bar{n}(\bar{x}_2) - \delta_{\bar{x}_1,\bar{x}_2}] \ldots [\bar{n}(\bar{x}_k) - \delta_{\bar{x}_1,\bar{x}_k} - \ldots - \delta_{\bar{x}_{k-1},\bar{x}_k}]\rangle .$$

„local factorial moments“

(3)

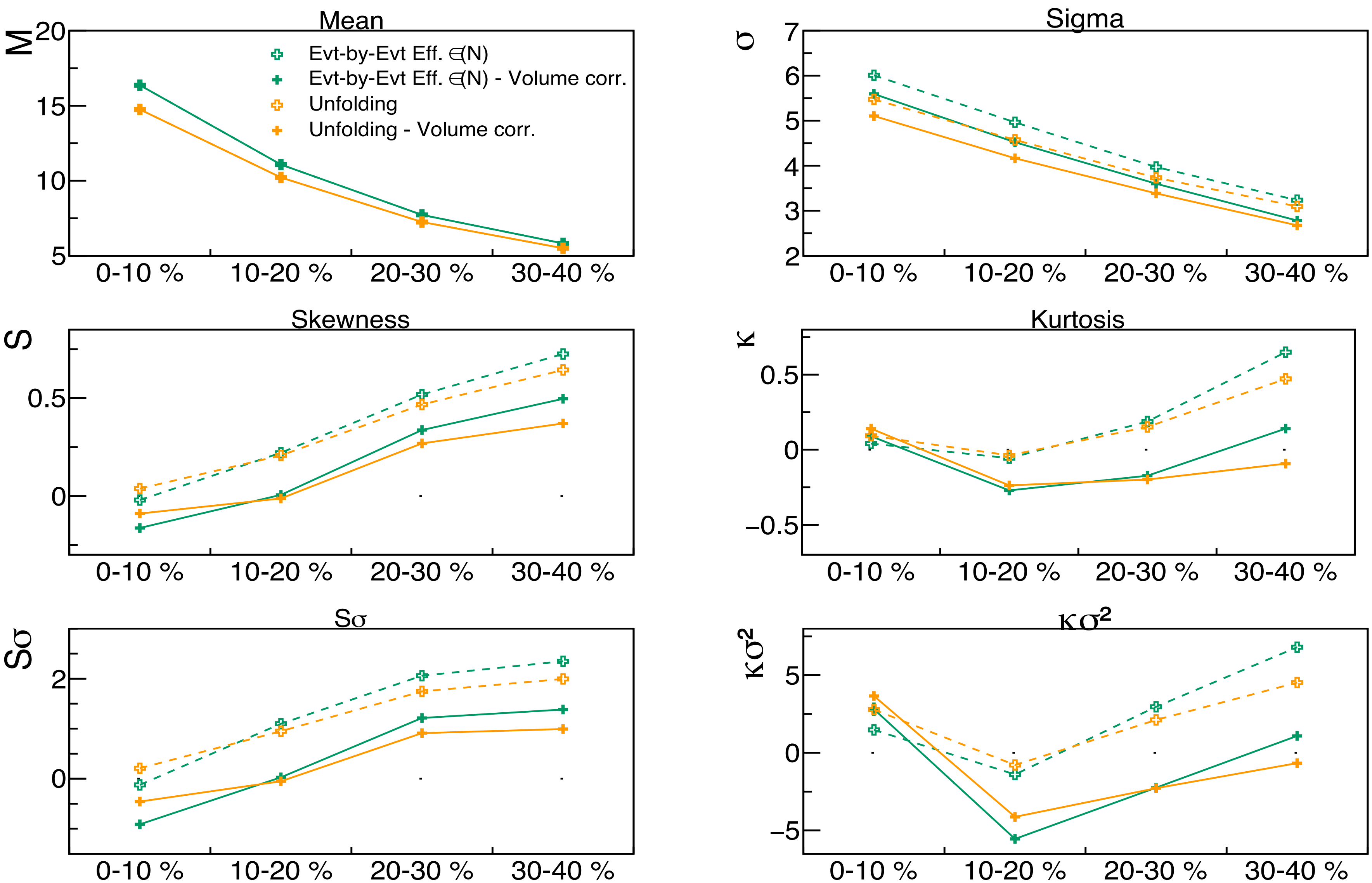
$$F_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} A_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k)$$

$$f_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k)$$

$$F_{i,k} = \sum_{x_1,\ldots,x_i} \sum_{\bar{x}_1,\ldots,\bar{x}_k} \frac{a_{i,k}(x_1, \ldots, x_i; \bar{x}_1, \ldots, \bar{x}_k)}{\epsilon(x_1) \ldots \epsilon(x_i) \bar{\epsilon}(\bar{x}_1) \ldots \bar{\epsilon}(\bar{x}_k)}$$

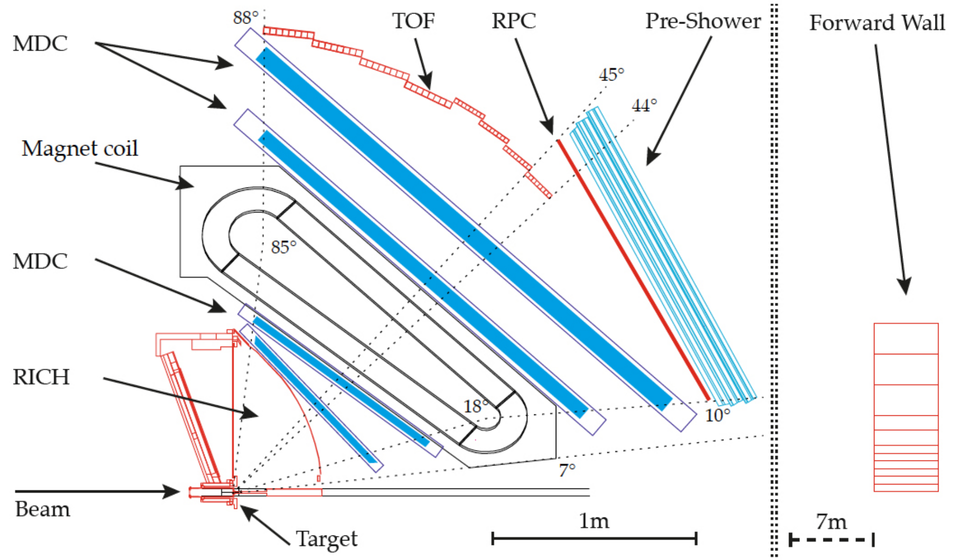
Bzdak & Koch, PRC 86 (2012);
Xiaofeng Luo, arXiv:1410.3914 (2014)

Volume Corrections



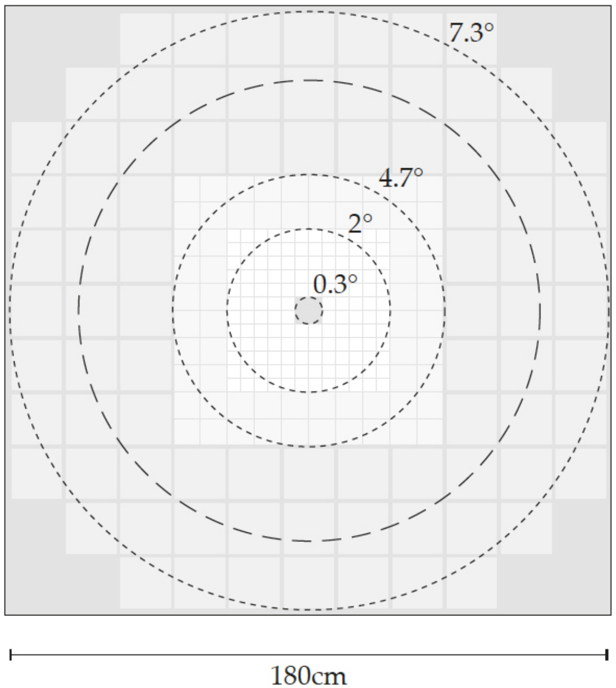
Centrality determination

- Number of tracks in MDCs
- Number of hits in time-of-flight detectors (TOF+RPC)
- Sum of charge in Forward Wall (spectators)

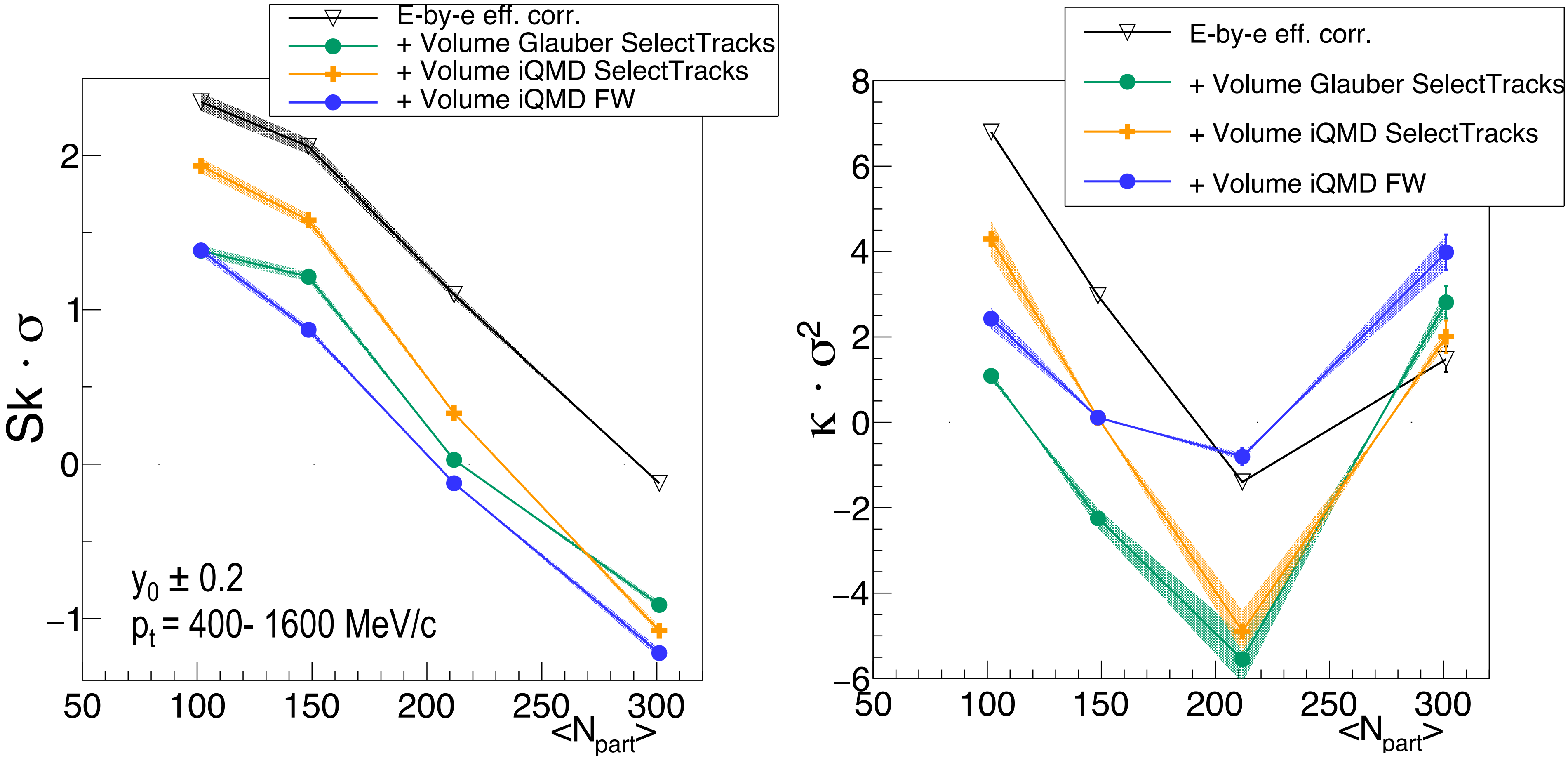


Forward Wall

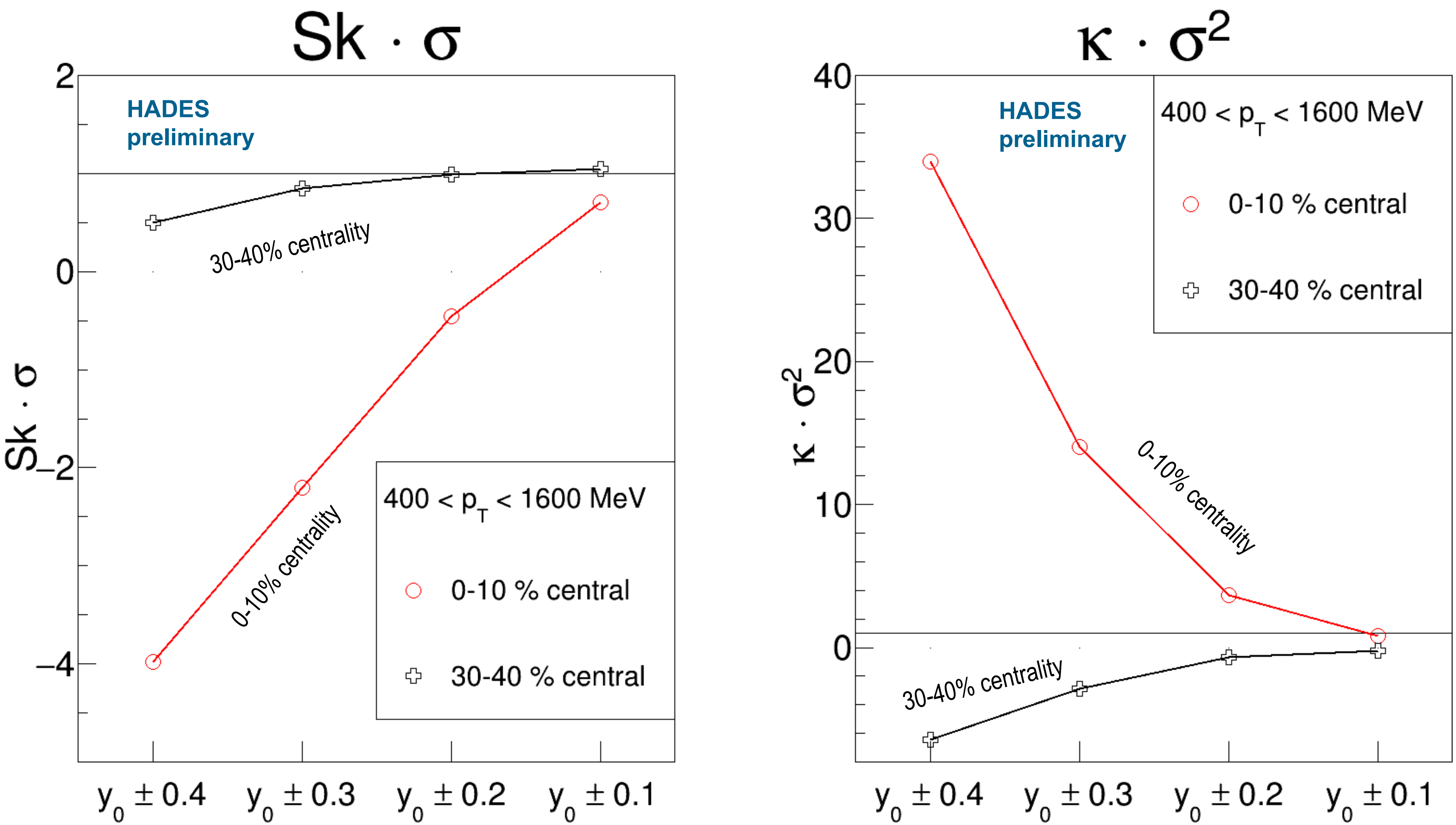
- Plastic Scintilator
- $\Theta = 0.5 - 7.5°$
- 3 different sizes



Volume Correction: Glauber vs IQMD

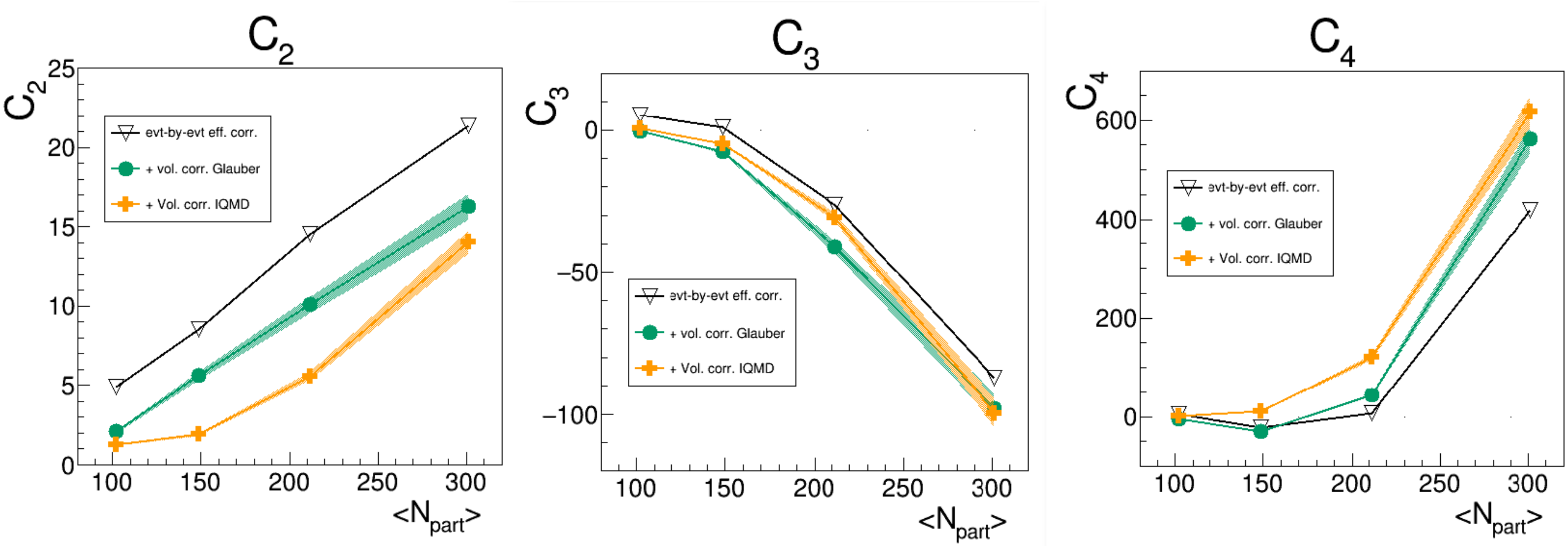


Poisson Limit: Δy→0



Proton n-particle correlations: C_n vs N_{part}

→ Going from cumulants κ_n to correlations C_n:



$y_0 \pm 0.2$
p_t = 400- 1600 MeV/c

→ C_4 is clearly the dominant contribution to the fourth order cumulant

Proton correlations C_n/N_p vs. N_{part}