2020/6/15 @ TCHoU

グラジエントフローによる QGP熱力学

Thermodynamics of QGP with the gradient flow

— Application of the SF*t*X method to lattice QCD at finite temperatures —

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Tomonaga Center for the History of the Universe

Gradient Flow

Narayanan-Neuberger (2006), Lüscher (2010-)

Gradient Flow

(example) Yang-Mills theory in the continuum

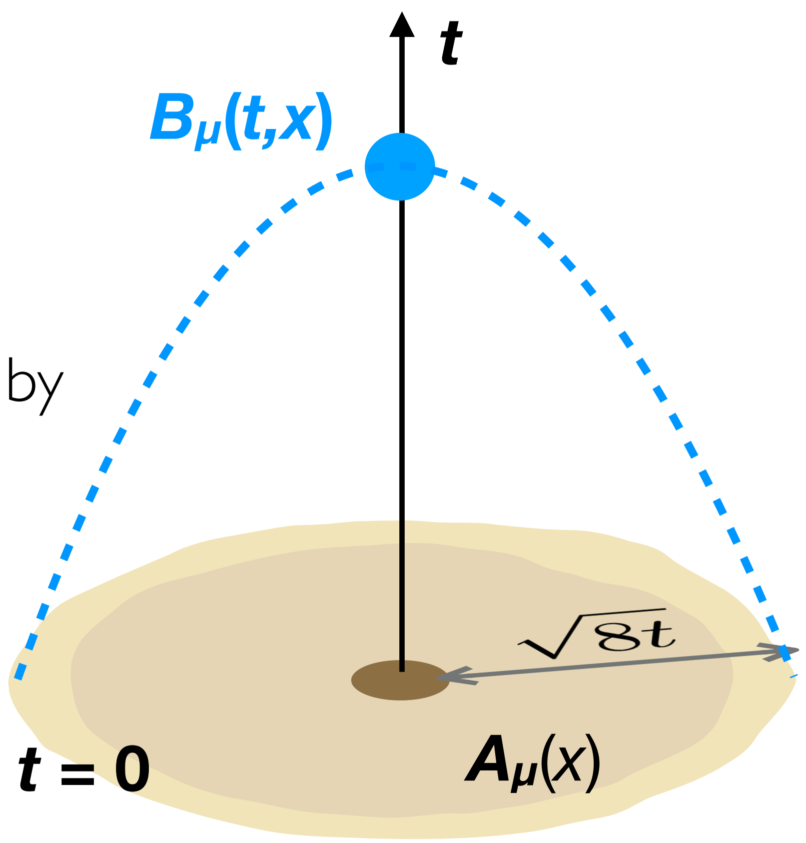
Original theory: gauge field $A_\mu(x)$ in D=4 dim. space-time,

$$S_{\rm YM}[A_\mu] = -\frac{1}{2g_0^2}\int d^Dx\, {\rm tr}[F_{\mu\nu}F_{\mu\nu}] = \frac{1}{2g_0^2}\int d^Dx\, F^a_{\mu\nu}F^a_{\mu\nu}$$

Introduce a fictitious "time" t, and evolve ("flow") the field A_μ by

$$\partial_t B_\mu(t,x) = -g_0^2 \frac{\delta S_{\rm YM}[B_\mu]}{\delta B_\mu} = D_\nu G_{\nu\mu}(t,x)$$

with $B_\mu(t=0,x) = A_\mu(x)$. $\quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$



This is a kind of diffusion equation.

Its perturbative solution reads

B_μ ~ smeared A_μ over a physical range of $\sqrt{(8t)}$.

("8" = 2 × D with D=4 dim.)

Quantum expectation values $\stackrel{\rm def.}{=}$ path-integration over the original fields A_μ

$$\langle B_\mu(t,x) B_\nu(s,y)\cdots\rangle \stackrel{\rm def.}{=} \frac{1}{Z}\int \mathcal{D}A_\mu\, B_\mu(t,x)B_\nu(s,y)\cdots e^{-S[A_\mu]}$$

Flowed operators are free from UV divergences and short-distance singularities.

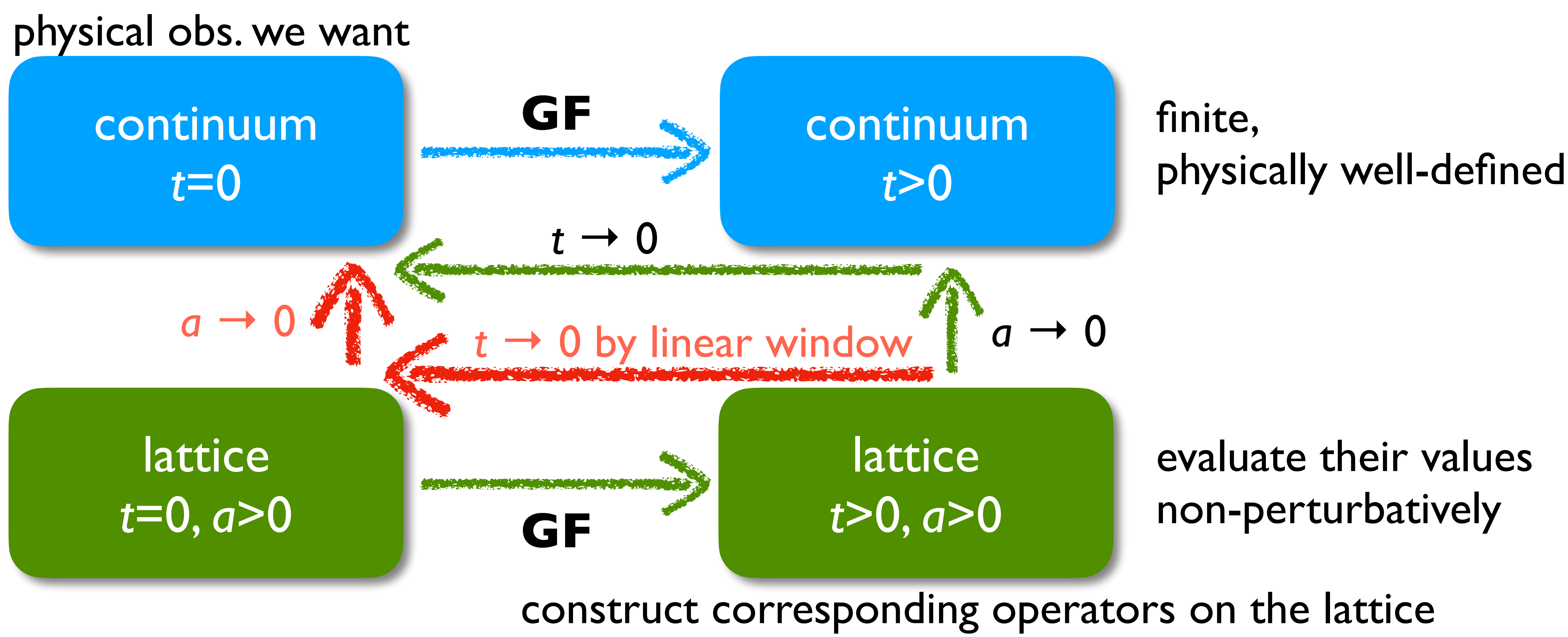
Lüscher-Weisz (2011)

SF*t*X method based on GF

H. Suzuki, PTEP 2013, 083B03 (2013) [E: 2015, 079201]

Making use of the finiteness of the GF, H. Suzuki developed **a general method to correctly calculate any renormalized observables non-perturbatively on the lattice**.

Small Flow-*t*ime eXpansion (SF*t*X) method

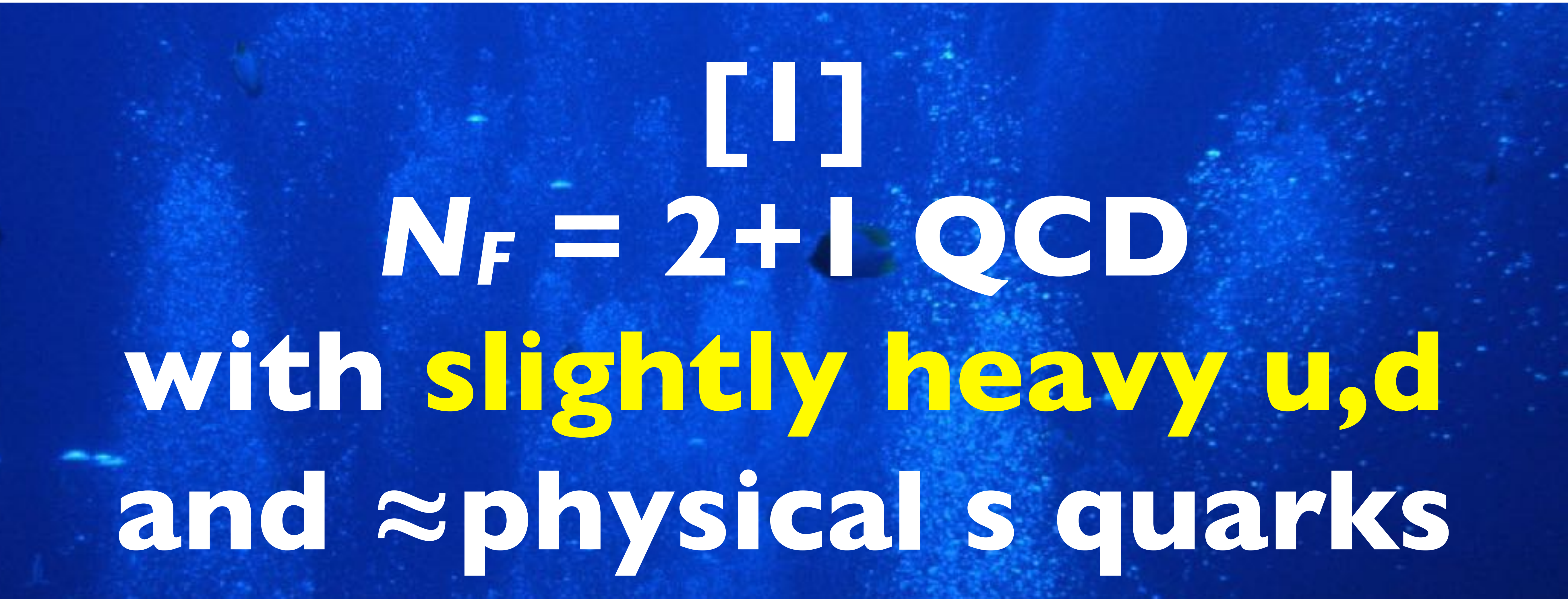


Because we can construct a lattice operator directly from the continuum operator,
this method is applicable also to observables whose base symmetry is broken on the lattice
(Poincaré inv., **chiral sym.**, etc.)

⇨ energy-momentum tensor

⇨ **QCD with Wilson-type quarks,** to cope with the problems due to chiral violation.

When we can identify a proper window, we may exchange the order of two extrapolations.



[1]
N_F = 2+1 QCD
with **slightly heavy u,d** and ≈physical s quarks

Taniguchi-Ejiri-Iwami-KK-Kitazawa-Suzuki-Umeda-Wakabayashi, Phys.Rev. D 96, 014509 (2017)

Taniguchi-KK-Suzuki-Umeda, Phys.Rev. D 95, 054502 (2017)

test of SF*t*X with dynamical quarks

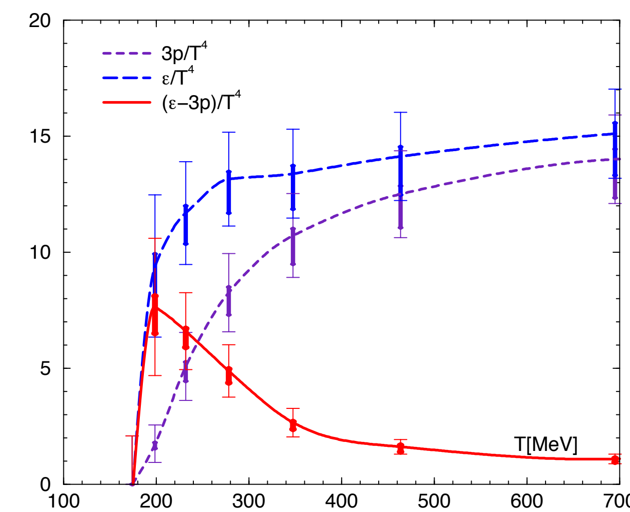
Taniguchi-Ejiri-Iwami-KK-Kitazawa-Suzuki-Umeda-Wakabayashi, Phys.Rev. D 96, 014509 (2017)

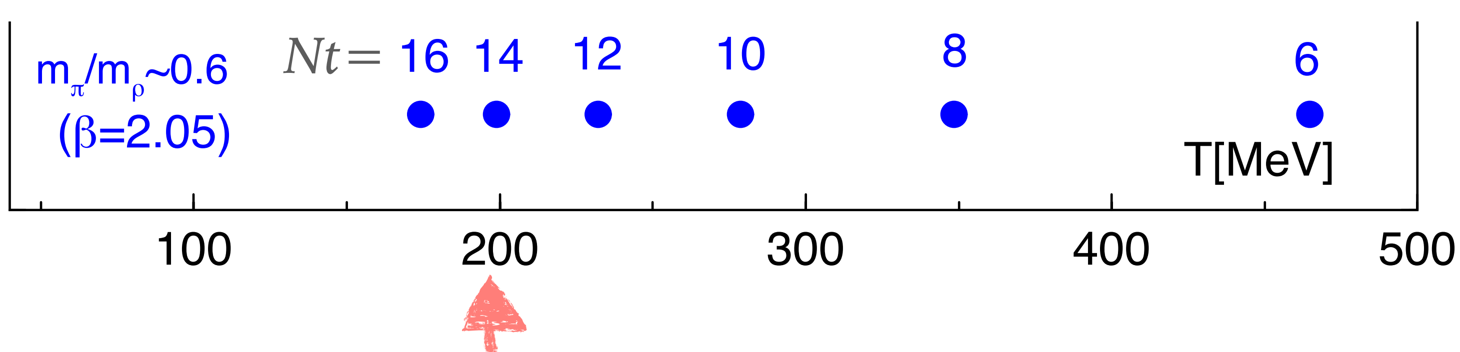
As the 1st step with dynamical quarks:

- Heavy ud quarks ($m_\pi/m_\rho \approx 0.63$) with ≈physical s quark ($m_{\eta ss}/m_\phi \approx 0.74$).
- Fine lattice ($a \approx 0.07$fm with improved action) using the fixed-scale approach.
- Compare with EoS by the conventional *T*-integration method.

WHOT-QCD Collab., Phys.Rev. D 85, 094508 (2012)

- ☑ N_f=2+1 QCD, RG-improved Iwasaki gauge + NP O(a)-improved Wilson quarks
- ☑ CP-PACS+JLQCD's $T=0$ config. ($\beta = 2.05$, 28^3x56, $a \approx 0.07$fm, $m_\pi/m_\rho \approx 0.63$):
  the lightest and the finest among the 3β x $5m_{ud}$ x $2m_s$ data points available.
- ☑ $T > 0$ by fixed-scale approach, WHOT-QCD config.(32^3xNt, Nt = 4, 6, 8, 10, 12, 14, 16)
- ☑ gauge measurements at every config.
- ☑ quark measurements every 10 config's, using a noisy estimator method.
- ☐ continuum extrapolation => to do





$T_{pc} \approx 190$ MeV

T (MeV)	T/T_{pc}	N_t	$t_{1/2}$	gauge confs.
0	0	56	24.5	650
174	0.92	16	8	1440
199	1.05	14	6.125	1270
232	1.22	12	4.5	1290
279	1.47	10	3.125	780
348	1.83	8	2	510
464	2.44	6	1.125	500
697	3.67	4	0.5	700

energy-momentum tensor

<u>In continuum</u>, EMT is defined as the generator of Poincaré transformation.

$$T_{\mu\nu} = \frac{2}{\sqrt{g}} \frac{\delta S}{\delta g_{\mu\nu}}\bigg|_{g_{\mu\nu}=\delta_{\mu\nu}} = \frac{1}{g_0^2}\left[F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma}\right]$$

for YM theory

- source of the gravity
- conserved Noether current associated with the Poincaré inv.
- a fundamental observable of the theory to extract
  - EoS (energy, pressute), momentum, shear stress, ...
  - fluctuation/correlation functions => specific heat, viscosity, ...

energy   momentum

$$\begin{pmatrix} T_{00} & T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{pmatrix}$$

shear stress   pressure

<u>**On the lattice**</u>, the Poincaré invariance is explicitly broken.

We have to

- fine-tune the **renormalization** and **mixing coefficients** of many operators

  to make the current conserved and to get the correct values of en. density etc. in the continuum limit.

Caracciolo et al., NP B309, 612 (1988); Ann.Phys. 197, 119 (1990)

$$\{T_{\mu\nu}\}_R(x) = \sum_{i=1}^{7} Z_i \mathcal{O}_{i\mu\nu}(x)|_{\text{lattice}} - \text{VEV},$$

where

$$\mathcal{O}_{1\mu\nu}(x) \equiv \sum_\rho F^a_{\mu\rho}(x)F^a_{\nu\rho}(x), \qquad \mathcal{O}_{2\mu\nu}(x) \equiv \delta_{\mu\nu}\sum_{\rho,\sigma} F^a_{\rho\sigma}(x)F^a_{\rho\sigma}(x),$$

$$\mathcal{O}_{3\mu\nu}(x) \equiv \bar\psi(x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\psi(x), \quad \mathcal{O}_{4\mu\nu}(x) \equiv \delta_{\mu\nu}\bar\psi(x)\overleftrightarrow{\not{D}}\psi(x),$$

$$\mathcal{O}_{5\mu\nu}(x) \equiv \delta_{\mu\nu} m_0 \bar\psi(x)\psi(x),$$

allowed by the lattice rotation symmetry =>

$$\mathcal{O}_{6\mu\nu}(x) \equiv \delta_{\mu\nu}\sum_\rho F^a_{\mu\rho}(x)F^a_{\mu\rho}(x), \qquad \mathcal{O}_{7\mu\nu}(x) \equiv \delta_{\mu\nu}\bar\psi(x)\gamma_\mu \overleftrightarrow{D}_\mu\psi(x)$$

GF with dynamical quarks

Lüscher, JHEP1304.123(2013)

For the finiteness, the flow action can be different from the original action as far as the gauge-covariance is preserved. To include quarks (matter fields), Lüscher proposed a simple method, in which the gauge flow is the same as the pure gauge case.

gauge flow

the same as the pure YM case

original gauge field at $t = 0$

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x)$$

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)],$$

$$D_\nu G_{\nu\mu}(t,x) = \partial_\nu G_{\nu\mu}(t,x) + [B_\nu(t,x), G_{\nu\mu}(t,x)],$$

quark flow

original quark field at $t = 0$

$$\partial_t \chi_f(t,x) = \Delta \chi_f(t,x), \qquad \chi_f(t=0,x) = \psi_f(x),$$

$$\partial_t \bar{\chi}_f(t,x) = \bar{\chi}_f(t,x) \overleftarrow{\Delta}, \qquad \bar{\chi}_f(t=0,x) = \bar{\psi}_f(x),$$

$$\Delta \chi_f(t,x) \equiv D_\mu D_\mu \chi_f(t,x), \qquad D_\mu \chi_f(t,x) \equiv [\partial_\mu + B_\mu(t,x)]\, \chi_f(t,x),$$

$$\bar{\chi}_f(t,x) \overleftarrow{\Delta} \equiv \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \overleftarrow{D}_\mu, \qquad \bar{\chi}_f(t,x) \overleftarrow{D}_\mu \equiv \bar{\chi}_f(t,x) \left[\overleftarrow{\partial}_\mu - B_\mu(t,x) \right]$$

only gauge fields involved

1) quark flow preserves the gauge and chiral symmetries.
   χ_f has the same gauge and chiral transformation properties as ψ_f.

2) quark flow is independent of spinor and flavor indices.
3) quark fields need renormalization <= can be handled numerically *a la* Makino-Suzuki

full QCD EMT by SFtX

Makino-Suzuki, PTEP 2014, 063B02 [E: 2015. 079202]

Measure flowed operators at $t \neq 0$:

$$\tilde{O}_{1\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x),$$
$$\tilde{O}_{2\mu\nu}(t,x) \equiv \delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x),$$
$$\tilde{O}^f_{3\mu\nu}(t,x) \equiv \varphi_f(t)\bar{\chi}_f(t,x)\left(\gamma_\mu \overleftrightarrow{D}_\nu + \gamma_\nu \overleftrightarrow{D}_\mu\right)\chi_f(t,x),$$
$$\tilde{O}^f_{4\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x),$$
$$\tilde{O}^f_{5\mu\nu}(t,x) \equiv \varphi_f(t)\delta_{\mu\nu}\bar{\chi}_f(t,x)\chi_f(t,x), \qquad \varphi_f(t) \equiv \frac{-6}{(4\pi)^2 t^2 \left\langle \bar{\chi}_f(t,x)\overleftrightarrow{\not{D}}\chi_f(t,x)\right\rangle_0}.$$

and combine them as

$$T_{\mu\nu}(x) = \lim_{t\to 0}\Big\{ c_1(t)\left[\tilde{O}_{1\mu\nu}(t,x) - \frac{1}{4}\tilde{O}_{2\mu\nu}(t,x)\right]$$
$$+ c_2(t)\left[\tilde{O}_{2\mu\nu}(t,x) - \left\langle \tilde{O}_{2\mu\nu}(t,x)\right\rangle_0\right]$$
$$+ c_3(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle \tilde{O}^f_{3\mu\nu}(t,x) - 2\tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right]$$
$$+ c_4(t)\sum_{f=u,d,s}\left[\tilde{O}^f_{4\mu\nu}(t,x) - \left\langle \tilde{O}^f_{4\mu\nu}(t,x)\right\rangle_0\right]$$
$$+ \sum_{f=u,d,s} c^f_5(t)\left[\tilde{O}^f_{5\mu\nu}(t,x) - \left\langle \tilde{O}^f_{5\mu\nu}(t,x)\right\rangle_0\right]\Big\},$$

$$c_1(t) = \frac{1}{\bar{g}(1/\sqrt{8t})^2} - \frac{1}{(4\pi)^2}\left[9(\gamma - 2\ln 2) + \frac{19}{4}\right]$$

etc.

Physical EMT extracted by $t\to 0$ extrapolation.

c_i : matching coefficients

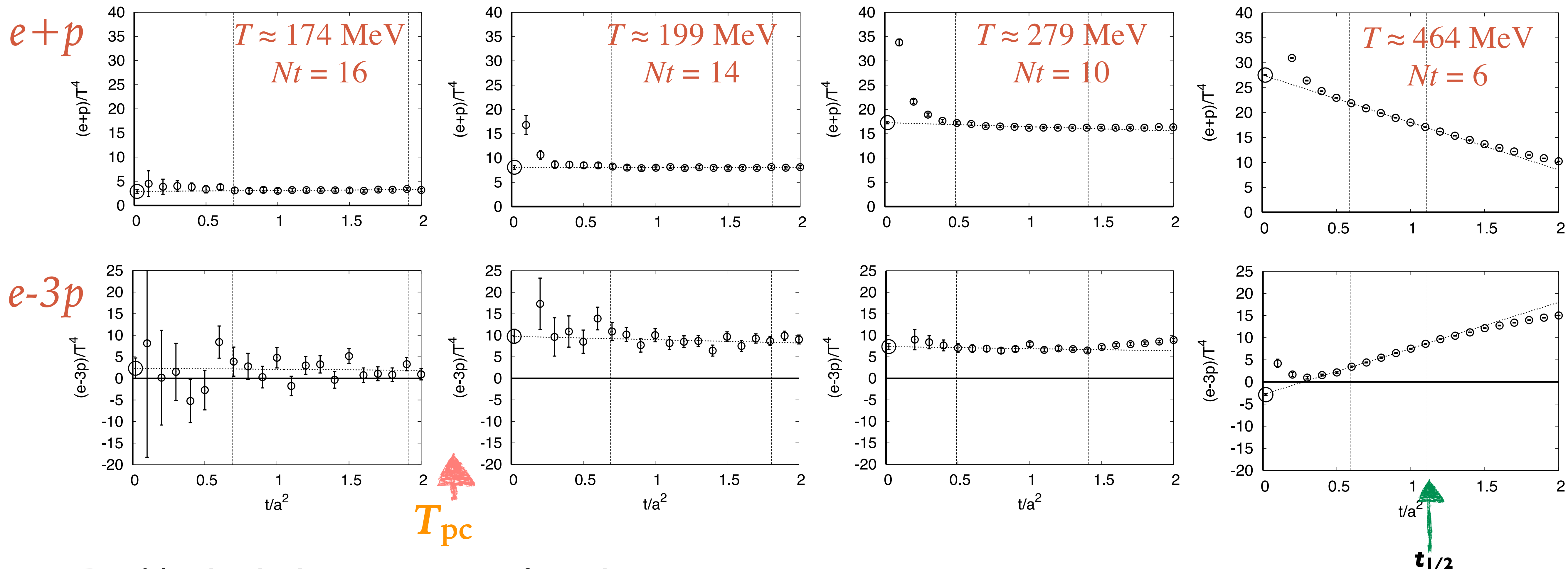
- to make $t\to 0$ smoother by removing known small-t mixings & t-dep. in the continuum
- to match the renormalization schemes when the observable is scheme-dependent
- perturbation theory applicable to calculate c_i at $t \approx 0$ in AF theories

In this study, we mainly use 1-loop c_i by Makino-Suzuki. We revisit the issue with 2-loop c_i later.

EMT with dynamical quarks

N_f=2+1 EMT with heavy u,d

$$\epsilon = -\langle T_{00} \rangle, \; p = \frac{1}{3}\sum_i \langle T_{ii} \rangle$$

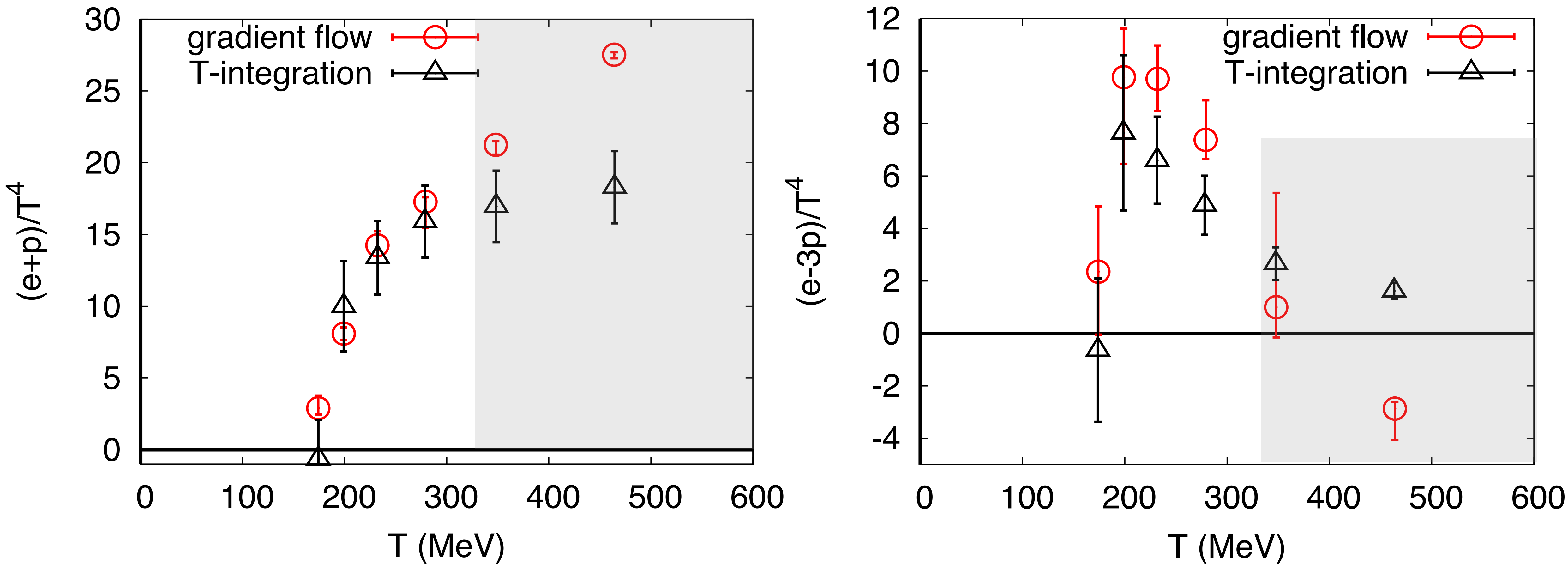


- a^2/t-like behavior at $t \approx 0$ visible.
- Linear behavior visible below $t_{1/2}$. (Nt=6 may be marginal.)
- a^2/t term looks negligible in the "linear windows" => Linear fit using the windows.
- At $T \approx 697$ MeV (Nt=4), no linear windows found.
- Smaller errors for e+p <= no T=0 subtraction required

EMT with dynamical quarks

Taniguchi-Ejiri-Iwami-KK-Kitazawa-Suzuki-Umeda-Wakabayashi, Phys.Rev. D 96, 014509 (2017)

N_f=2+1 EoS with heavy u,d

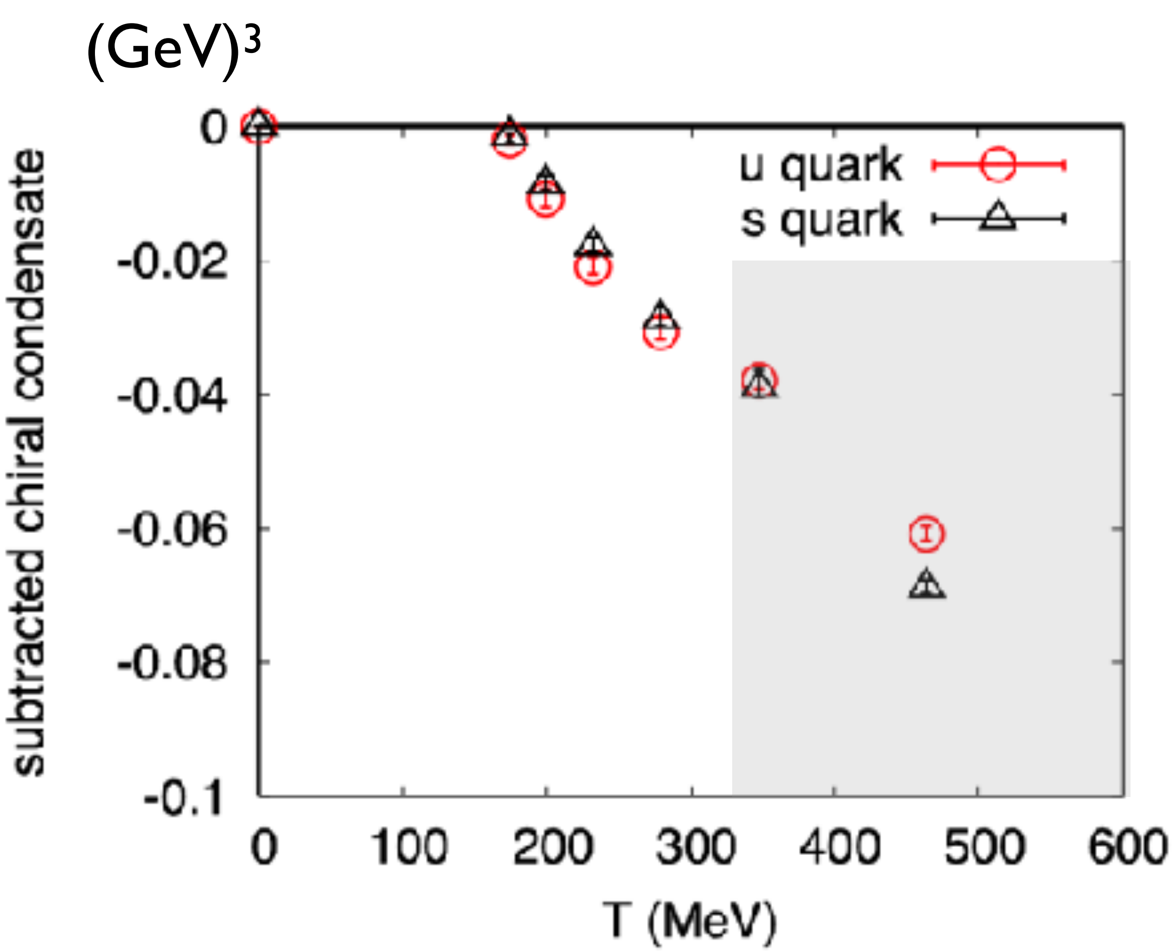


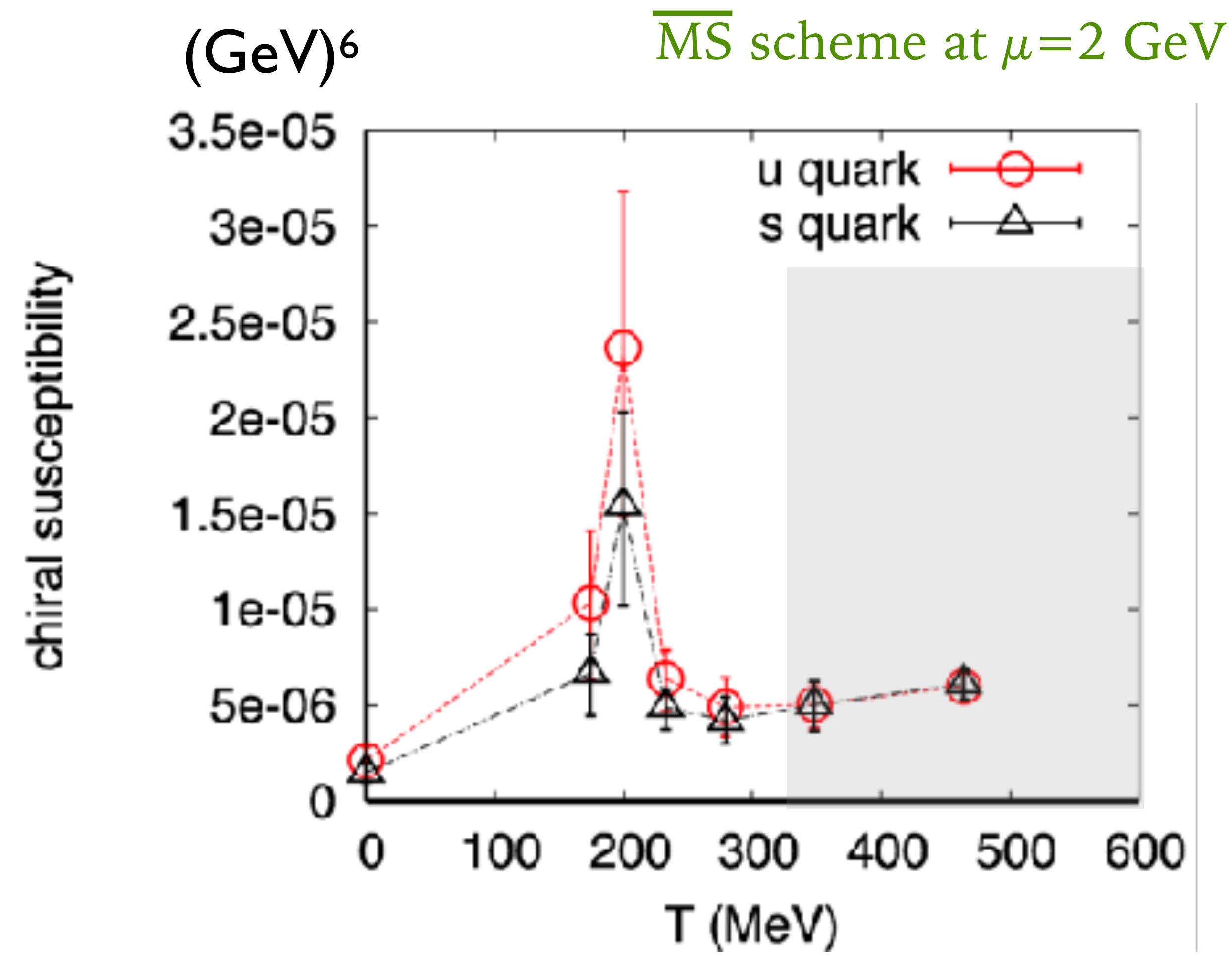
- EoS by SF*t*X agrees with conventional method at $T \leq 300$ MeV ($Nt \geq 10$). Suggest $a \approx 0.07$fm close to the cont. limit.
- Disagreement at $T \geq 350$ MeV due to $O((aT)^2 = 1/Nt^2)$ lattice artifact at $Nt \lesssim 8$. [Note that this lattice artifact is independent of *a*.]

chiral condensate / susceptibility

Taniguchi-Ejiri-Iwami-KK-Kitazawa-Suzuki-Umeda-Wakabayashi, Phys.Rev. D 96, 014509 (2017)

N_f=2+1 chiral cond. / disconnected susceptibility





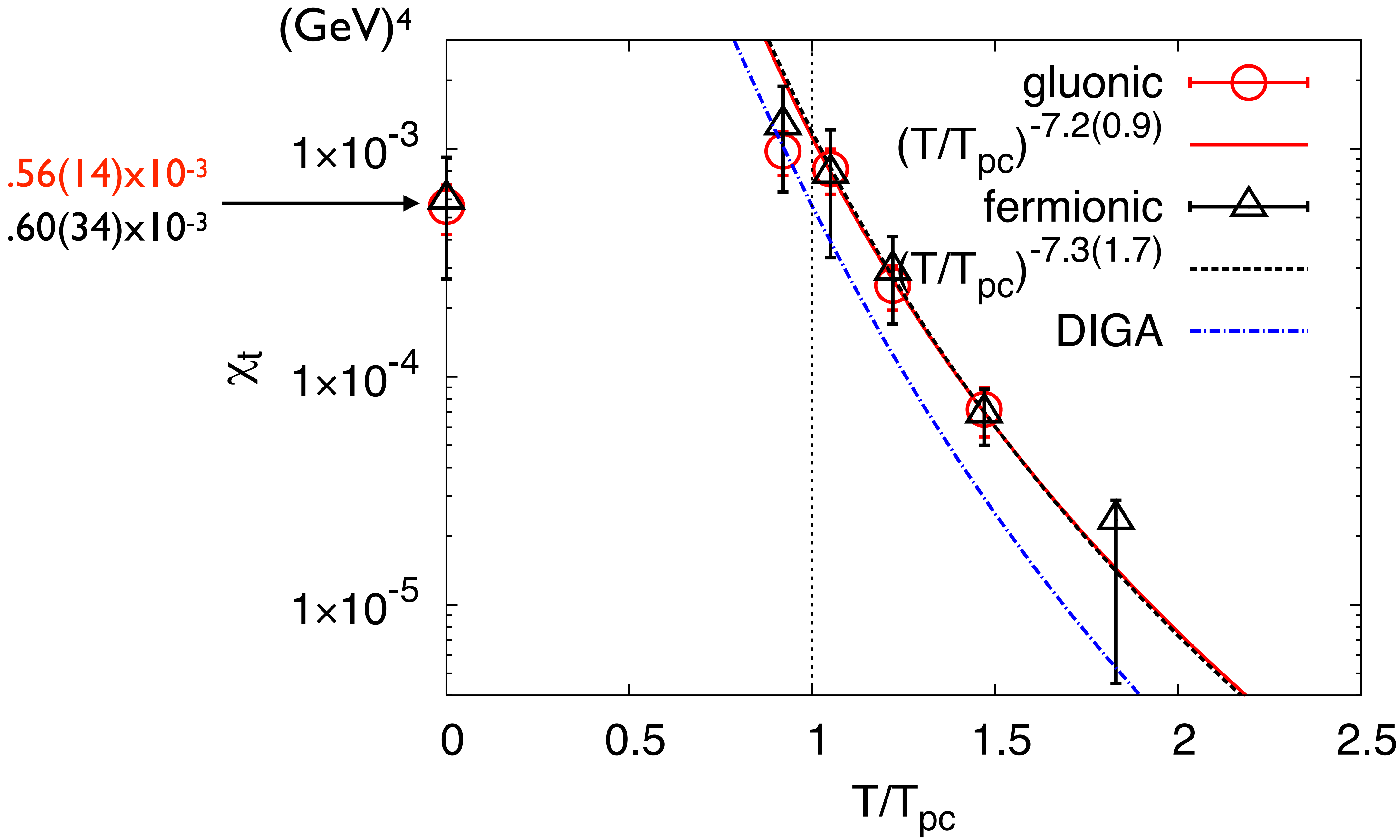
- ☑ Crossover suggested around $T_{pc}\approx$190 MeV, consistent with previous study.
- ☑ Peak higher with decreasing m_q, as expected.

=> Physically expected results even with Wilson-type quarks!

SF*t*X powerful to extract physical properties.

topological susceptibility

Taniguchi-KK-Suzuki-Umeda, Phys.Rev. D 95, 054502 (2017)



- Two definitions agree well => SF*t*X enables us reliable predictions.
- Power low consistent with a prediction of Dilute Instanton Gas model.

[1A]
Issue of renormalization-scale in N_F = 2+1 QCD with slightly heavy u,d

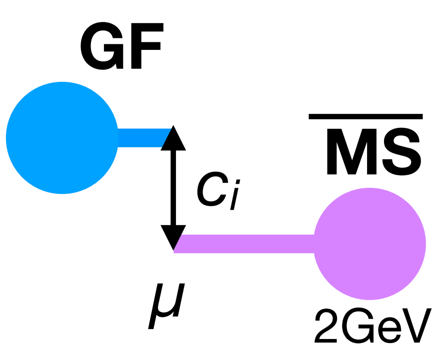
Taniguchi-Ejiri-KK-Kitazawa-Suzuki-Umeda, arXiv: 2005.00251 (2020)

renormalization scale *μ*

- matching coefficients of the SF*t*X method

$$c_1(t) = \frac{1}{g^2}\left(1 + \frac{g^2}{(4\pi)^2}\left[-\beta_0 L(\mu,t) - \frac{7}{3}C_A + \frac{3}{2}T_F\right] + \frac{g^4}{(4\pi)^4}\left\{-\beta_1 L(\mu,t) + C_A^2\left(-\frac{14482}{405} - \frac{16546}{135}\ln 2 + \frac{1187}{10}\ln 3\right) + C_A T_F\left[\frac{59}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{10873}{810} + \frac{73}{54}\pi^2 - \frac{2773}{135}\ln 2 + \frac{302}{45}\ln 3\right] + C_F T_F\left[-\frac{256}{9}\mathrm{Li}_2\left(\frac{1}{4}\right) + \frac{2587}{108} - \frac{7}{9}\pi^2 - \frac{106}{9}\ln 2 - \frac{161}{18}\ln 3\right]\right\}\right)$$

etc. with $L(\mu,t) \equiv \ln\left(2\mu^2 t\right) + \gamma_E$



Harlander-Kluth-Lange, EPJC 78:944 (2018)

c_i at small t are calculated in terms of the MS-bar running coupling $g(\mu)$ and mass $m(\mu)$.
The MS-bar renorm. scale μ is free to choose, as far as the perturbative expansions are OK.
Final results should be indep. of μ.

A conventional choice is $\mu(t) = \mu_d(t) \equiv \frac{1}{\sqrt{8t}}$, a natural scale of flowed operators.

HKL suggested $\mu_0(t) \equiv \frac{1}{\sqrt{2e^{\gamma_E}t}}$ which makes $L(\mu,t) = 0$ and suppresses NNLO in a similar level as μ_d.

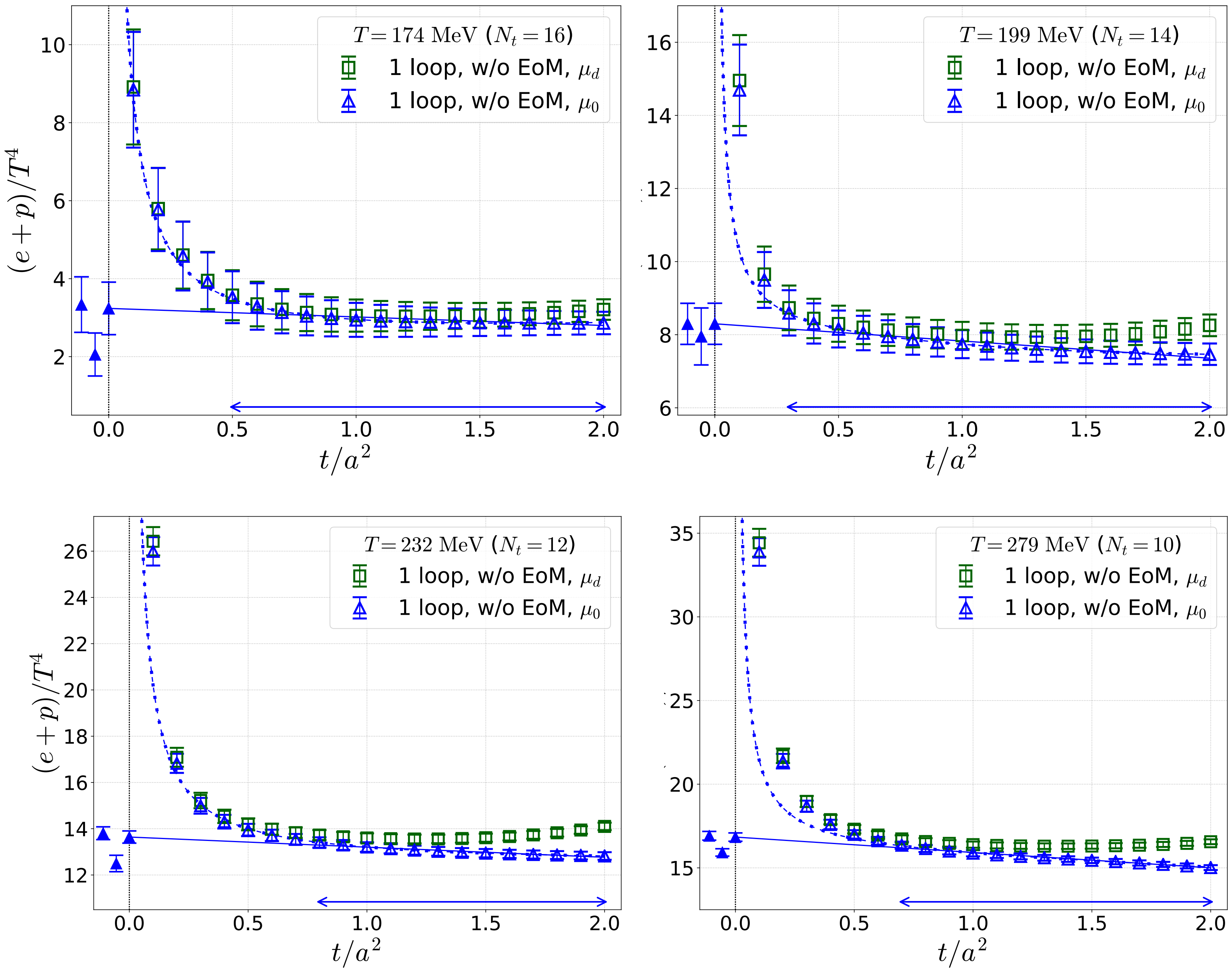
Practically $\mu_0(t) \approx 1.5\ \mu_d(t)$ => μ_0 more perturbative
extends the perturbative region towards larger t

[A larger $\mu(t)$ is even more perturbative, but a huge $L(\mu,t)$ breaks the perturbative expansion.]

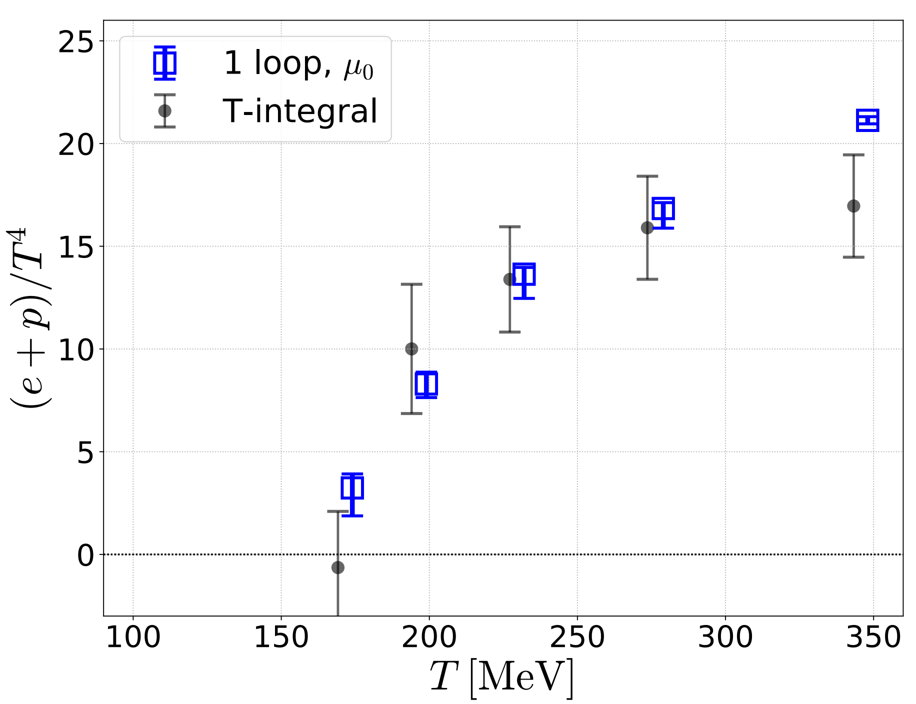
EoS with heavy u,d

Taniguchi-Ejiri-KK-Kitazawa-Suzuki-Umeda, arXiv: 2005.00251 (2020)

entropy density $(e+p)/T^4$



Results with the μ_0-scale

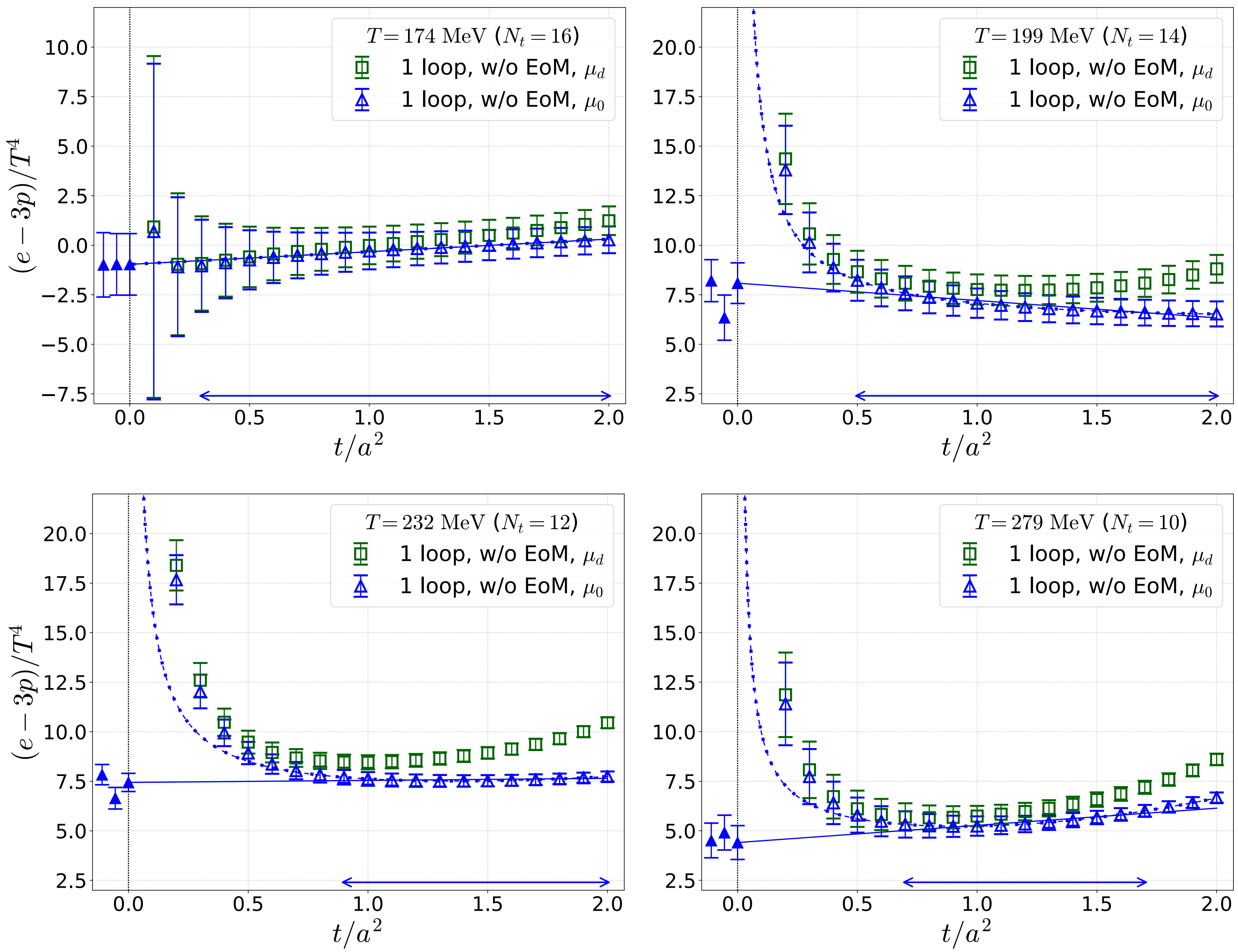


☑ μ_0 and μ_d results consistent with each other

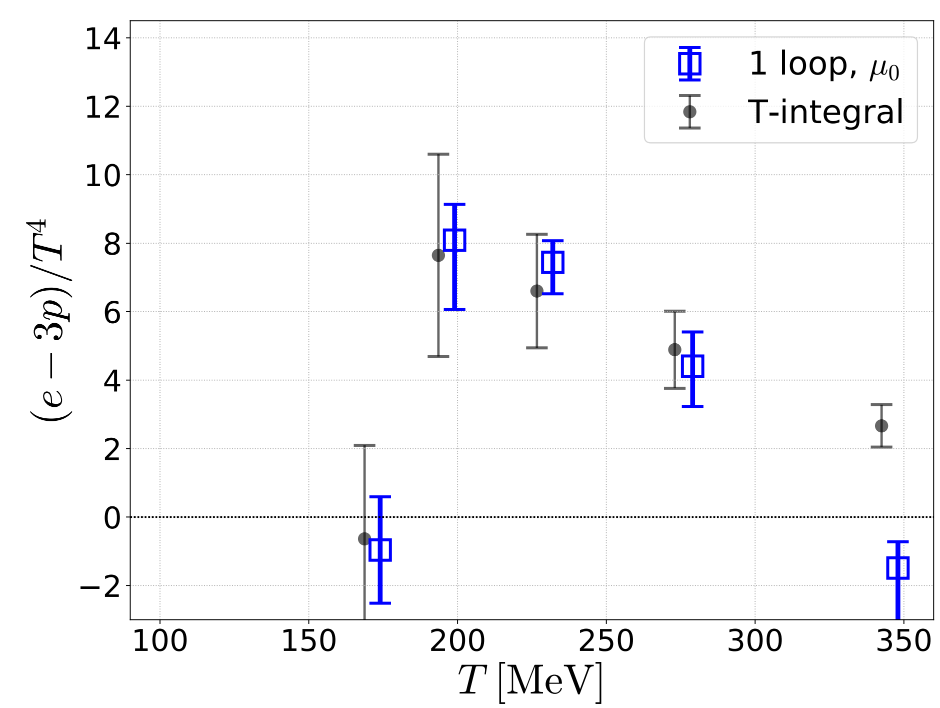
EoS with heavy u,d

Taniguchi-Ejiri-KK-Kitazawa-Suzuki-Umeda, arXiv: 2005.00251 (2020)

trace anomaly (e-3p)/T⁴



Results with the μ_0-scale



☑ μ_0 and μ_d results consistent with each other

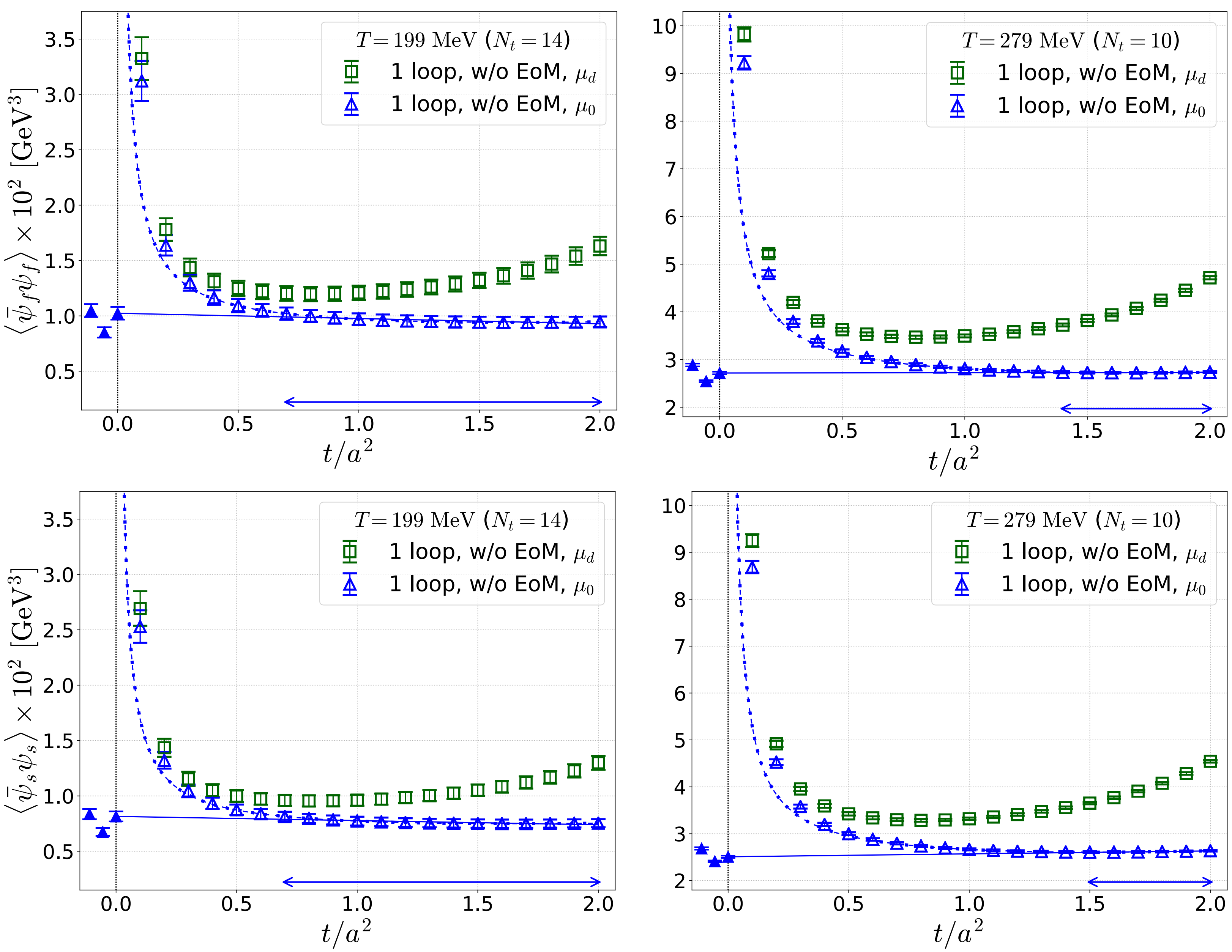
chiral condensate with heavy u,d

Taniguchi-Ejiri-KK-Kitazawa-Suzuki-Umeda, arXiv: 2005.00251 (2020)

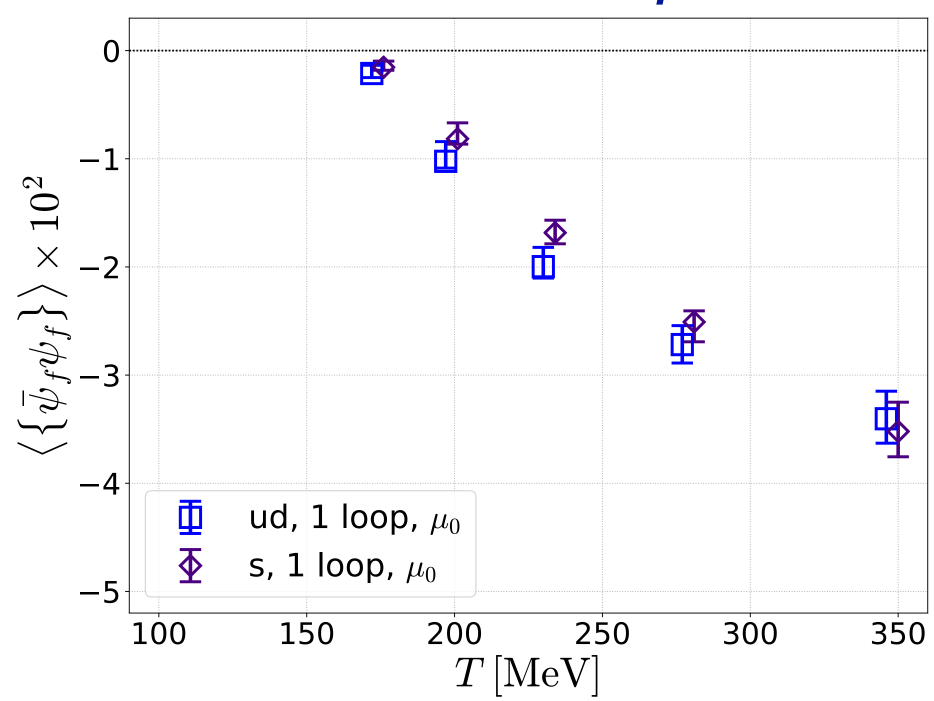
ud- and s-chiral cond. (VEV-subtracted)

$-\langle\{\bar{\psi}_u\psi_u\}\rangle_{\overline{\mathrm{MS}}}(\mu=2\,\mathrm{GeV})$

in unit of GeV^3



Results with the μ_0-scale

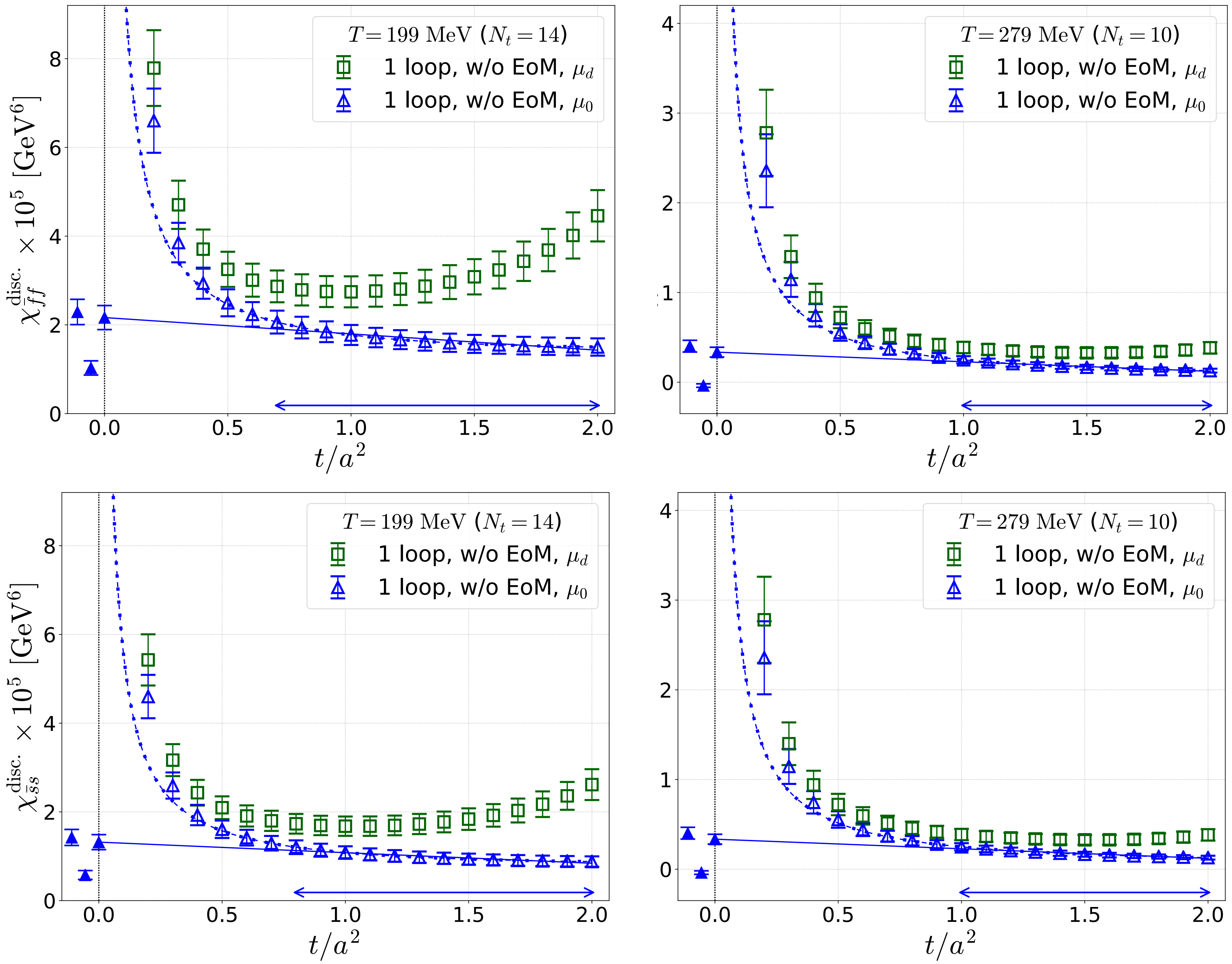


- ☑ μ_0 and μ_d results consistent with each other
- ☑ μ_0 improves linear behavior at large t => more reliable linear extrapolations

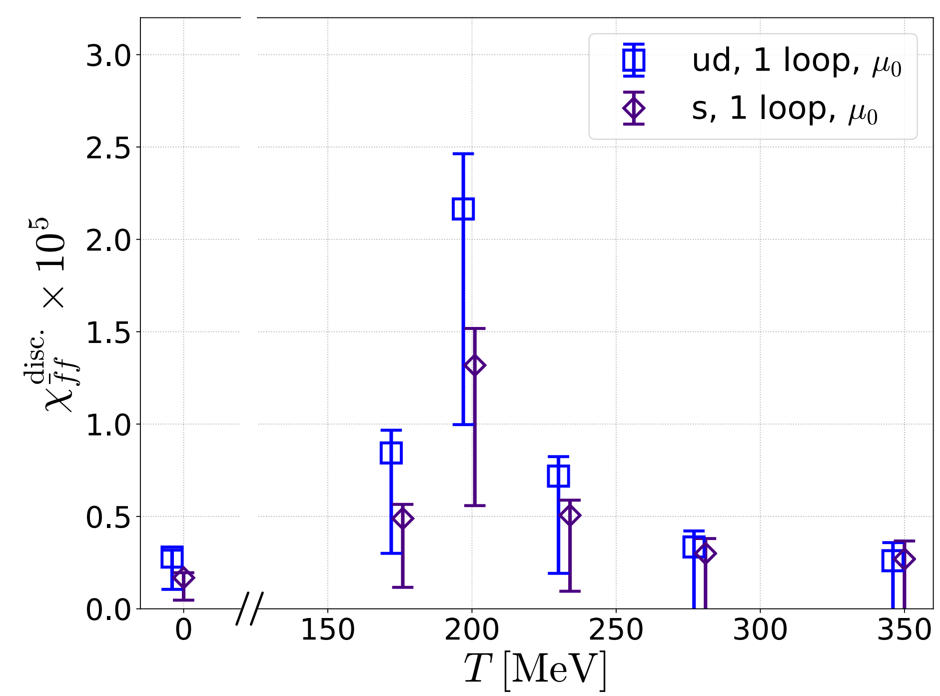
chiral susceptibility with heavy u,d

Taniguchi-Ejiri-KK-Kitazawa-Suzuki-Umeda, arXiv: 2005.00251 (2020)

ud- and s-chiral suscept. (disconnected)



Results with the μ_0-scale



☑ μ_0 and μ_d results consistent with each other

☑ μ_0 improves linear behavior at large *t*

=> μ_0 extend the reliability/applicability of the SF*t*X method => helps the phys. pt. study

[2]
N_F = 2+1 QCD
with **physical** u,d,s quarks

KK-Baba-SuzukiA-Ejiri-Kitazawa-SuzukiH-Taniguchi-Umeda, PoS Lattice2019, 088 (2020)

(2+1)-flavor **phys.pt.** QCD

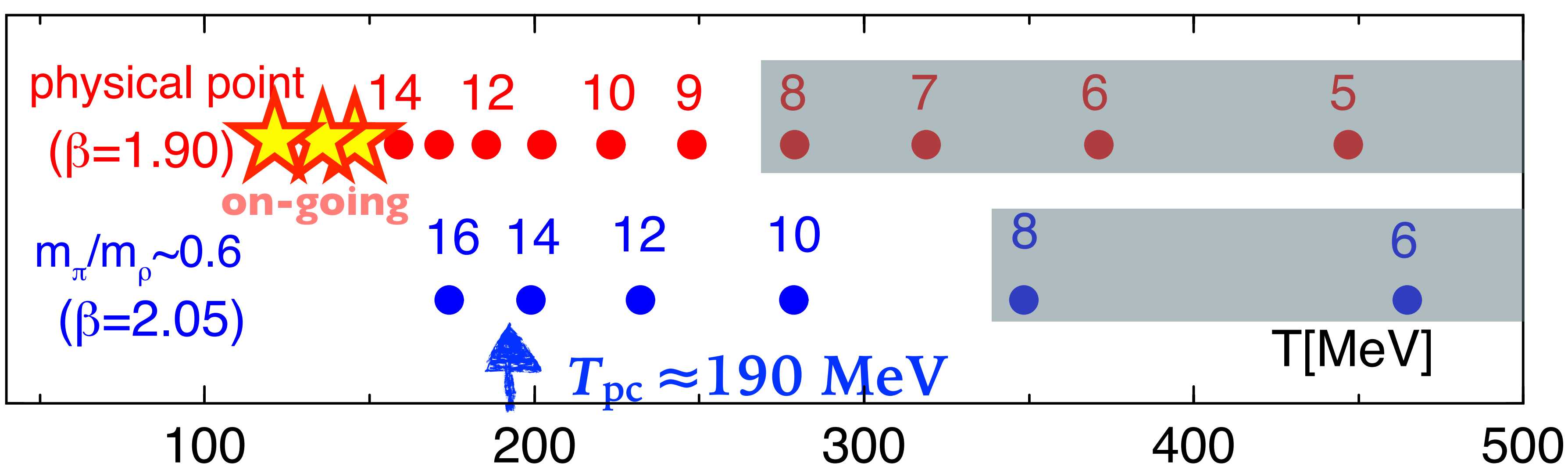
WHOT-QCD, EPJ Conf. 175, 07023 (2018)

+ New data at T≈122–146 MeV (prelim.)

- RG-improved Iwasaki gauge + NP O(a)-improved Wilson quarks
- T=0 configs. of PACS-CS (β=1.9, 32^3x64, ***a*≈0.09fm**) [Phys.Rev.D79, 034503 (2009)] 80 configs.
- All quarks fine-tuned to the **phys.pt.** by reweighting [Phys.Rev.D81, 074503 (2010)] using m_π, m_K, m_Ω inputs.
- T>0 by fixed-scale approach, (32^3xNt, Nt = 4, 5, ... , 18): **T≈122 – 549 MeV.**

  Odd Nt too, to have a finer T-resolution.

  Generated directly at the phys.pt. w/o reweighting [β=1.9, Kud=0.13779625, Ks=0.13663377].



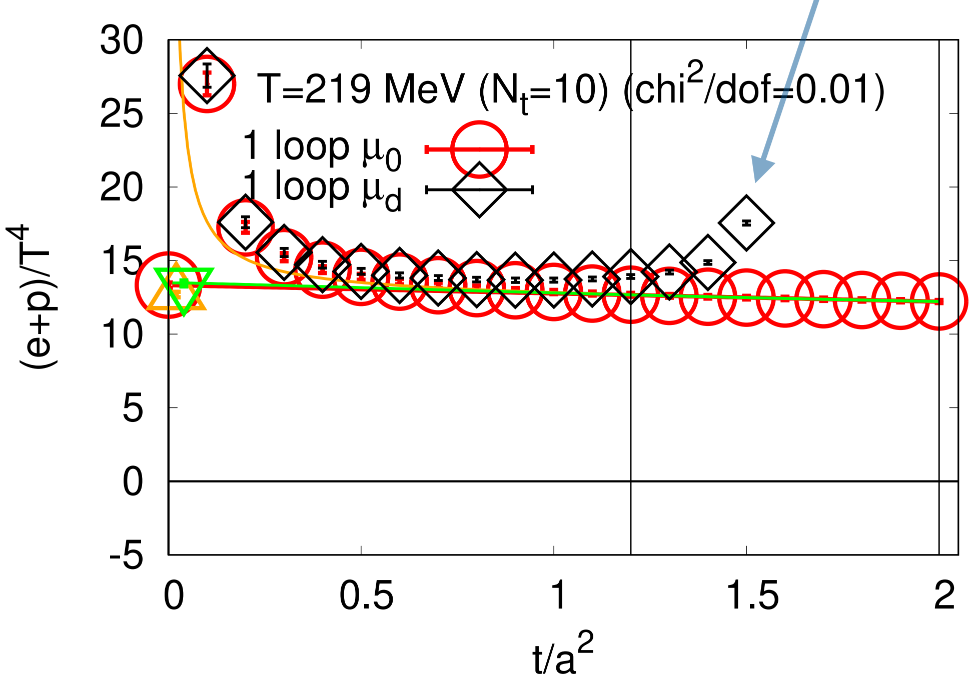
T[MeV]	T/T_{pc}	N_t	$t_{1/2}$	gauge confs.	fermion confs.
0	0	64	32	80	80
122		18	10.125	308	308
129		17	9.03125		
137		16	8	239	239
146		15	7.03125	143	143
157		14	6.125	650	65
169		13	5.28125	550	55
183		12	4.5	610	61
199		11	3.78125	890	89
219		10	3.125	690	69
244		9	2.53125	780	78
274		8	2	680	68
313		7	1.53125	220	22
366		6	1.125	280	280
439		5	0.78125	130	130
548		4	0.5	70	70

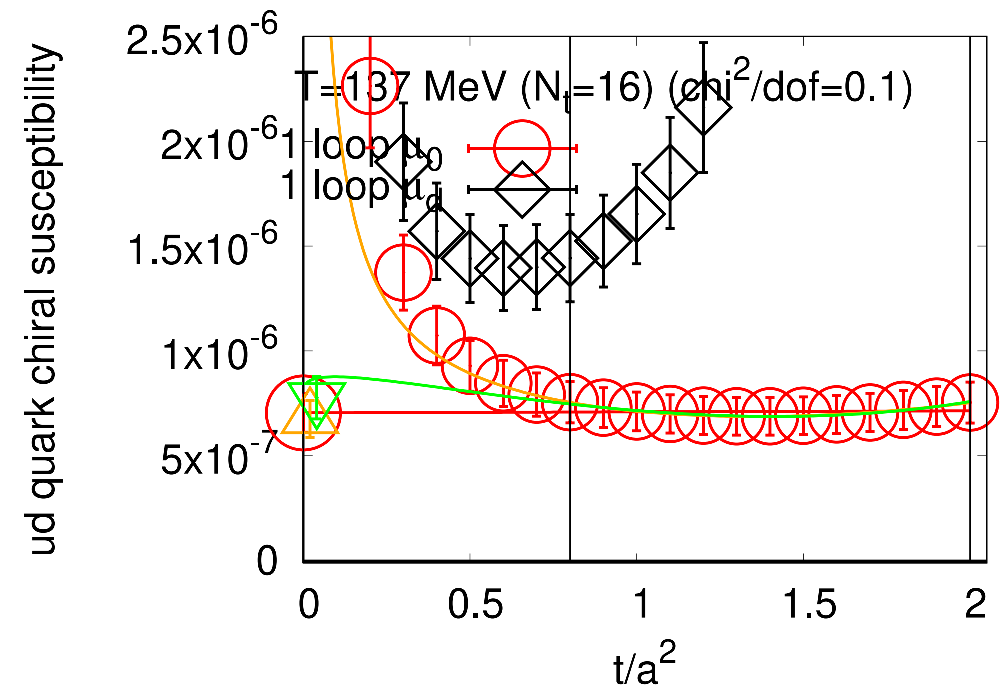
on-going

- Where is T_{pc} for physical m_q? Expect T_{pc}^{phys} < 190 MeV.
- Lattice is slightly coarser than the heavy QCD case (a≈0.07fm).
- Expect a-indep. lattice artifacts of O($(aT)^2$=$1/N_t^2$) at N_t ≤ 8 (T≥274 MeV)

renormalization scale μ

- ☐ Lattice at $a \approx 0.09$fm is slightly coarser than the heavy QCD case ($a \approx 0.07$fm).
  => Perturbative behavior worse --- μ_0 may help.
  $g(\mu(t))$ becomes large at $t/a^2 \approx 1.5$ with $\mu_d(t)$, but remains small up to ≈ 3 with $\mu_0(t)$.





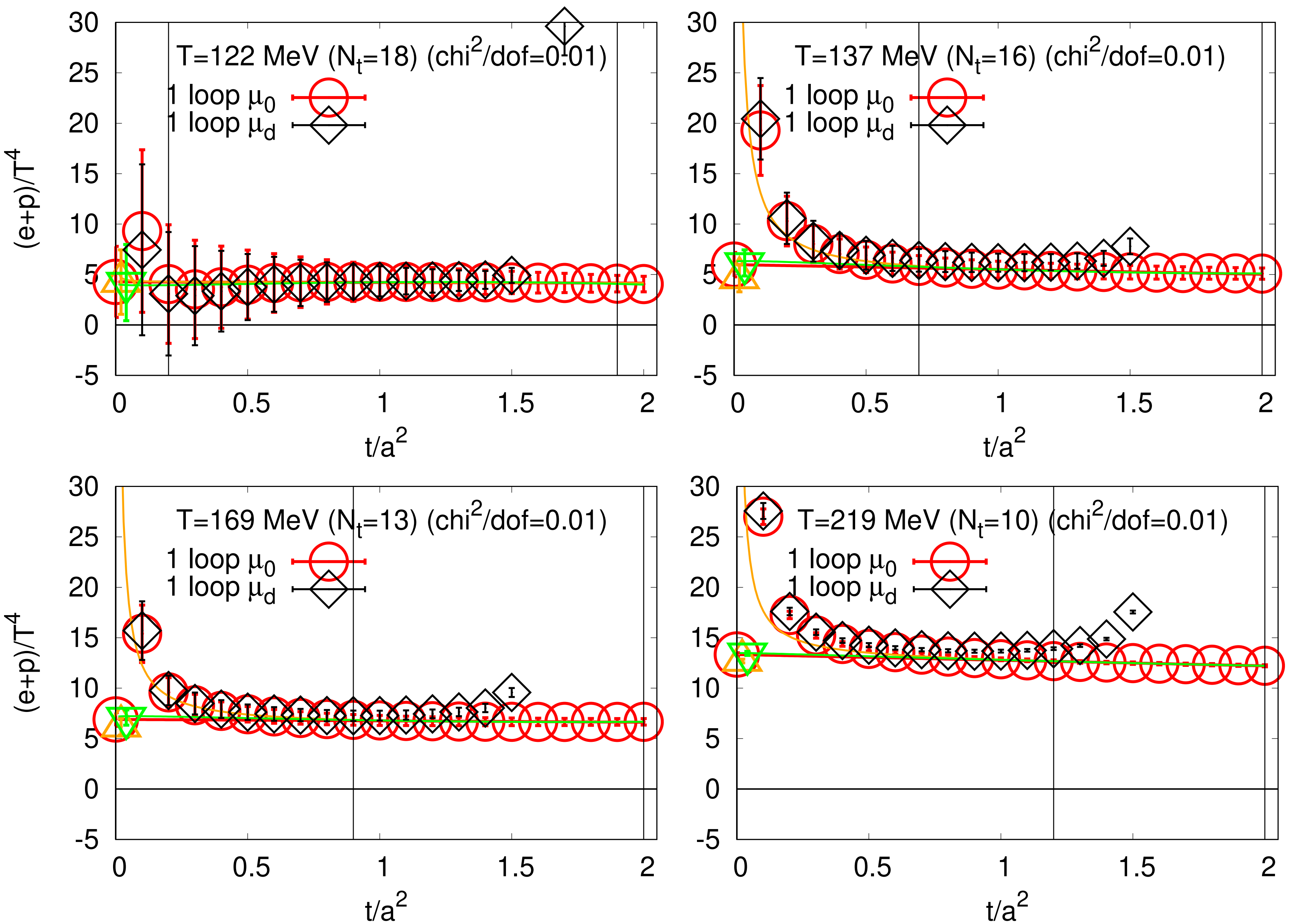
- ☑ μ_0 and μ_d results consistent with each other
- ☑ μ_0 improves linear behavior at large t
  => μ_0 extend the reliability/applicability of the SFtX method

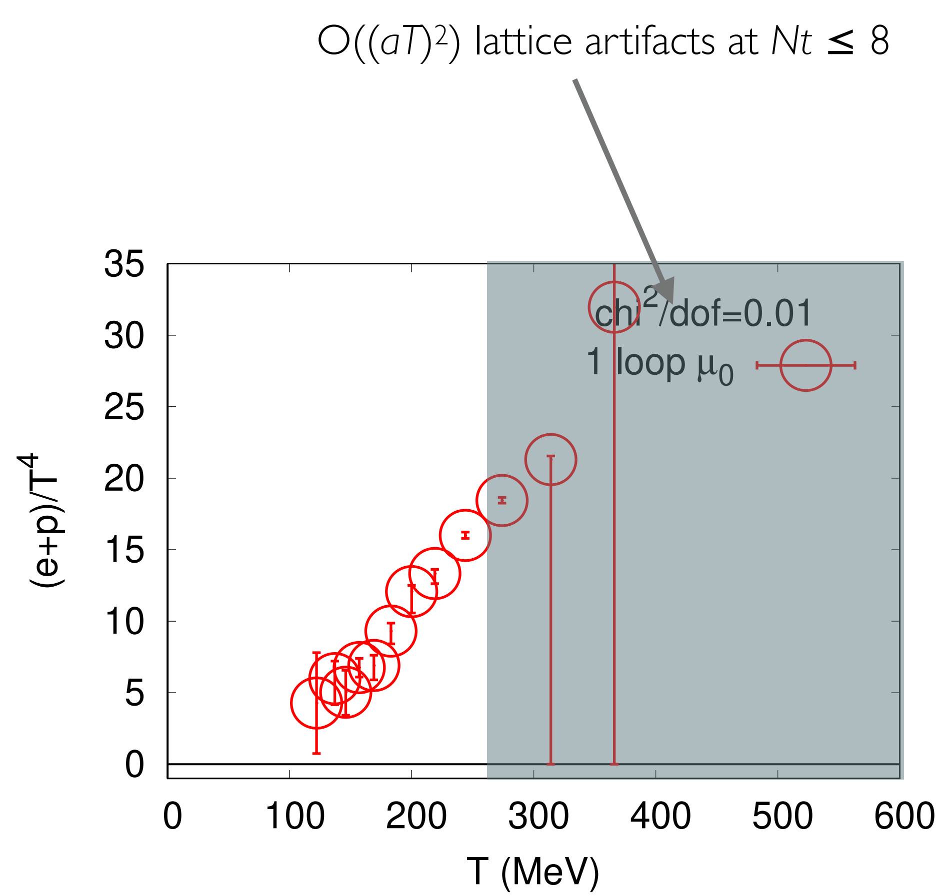
EoS at the physical point

entropy density $(e+p)/T^4$

1-loop
μ_0-scale

Preliminary

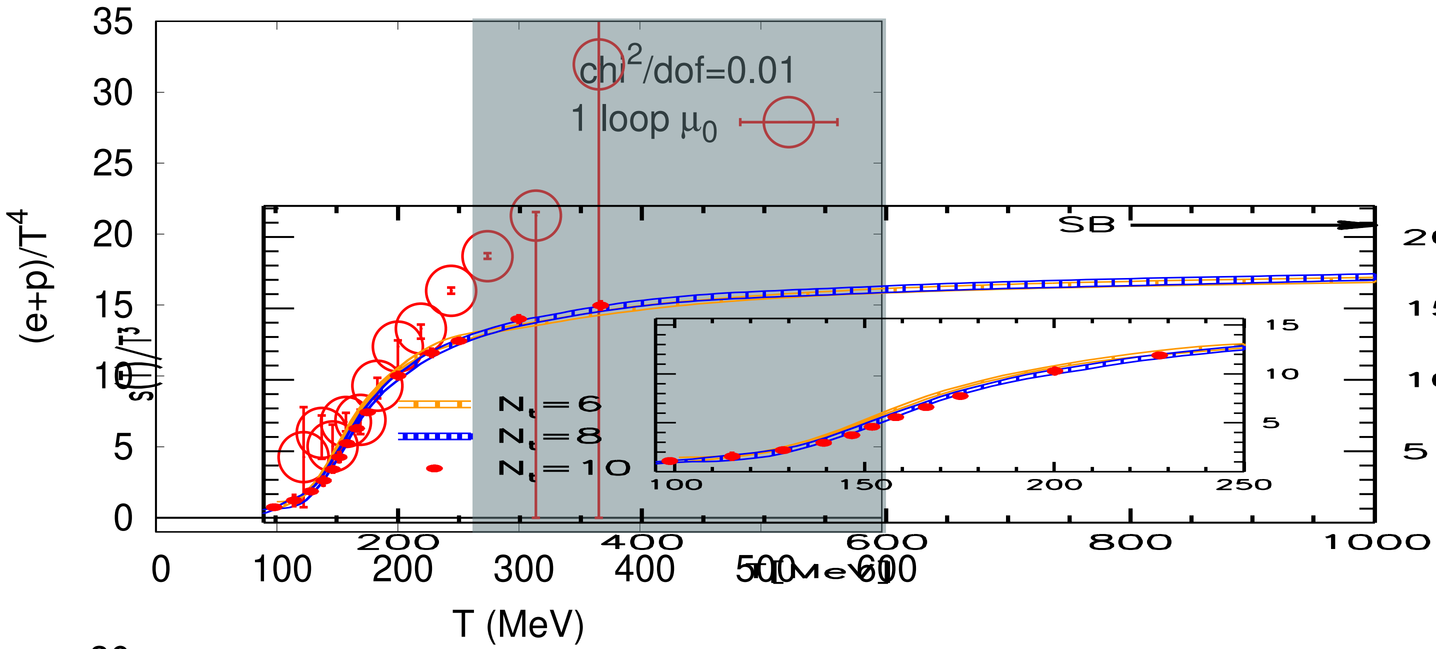




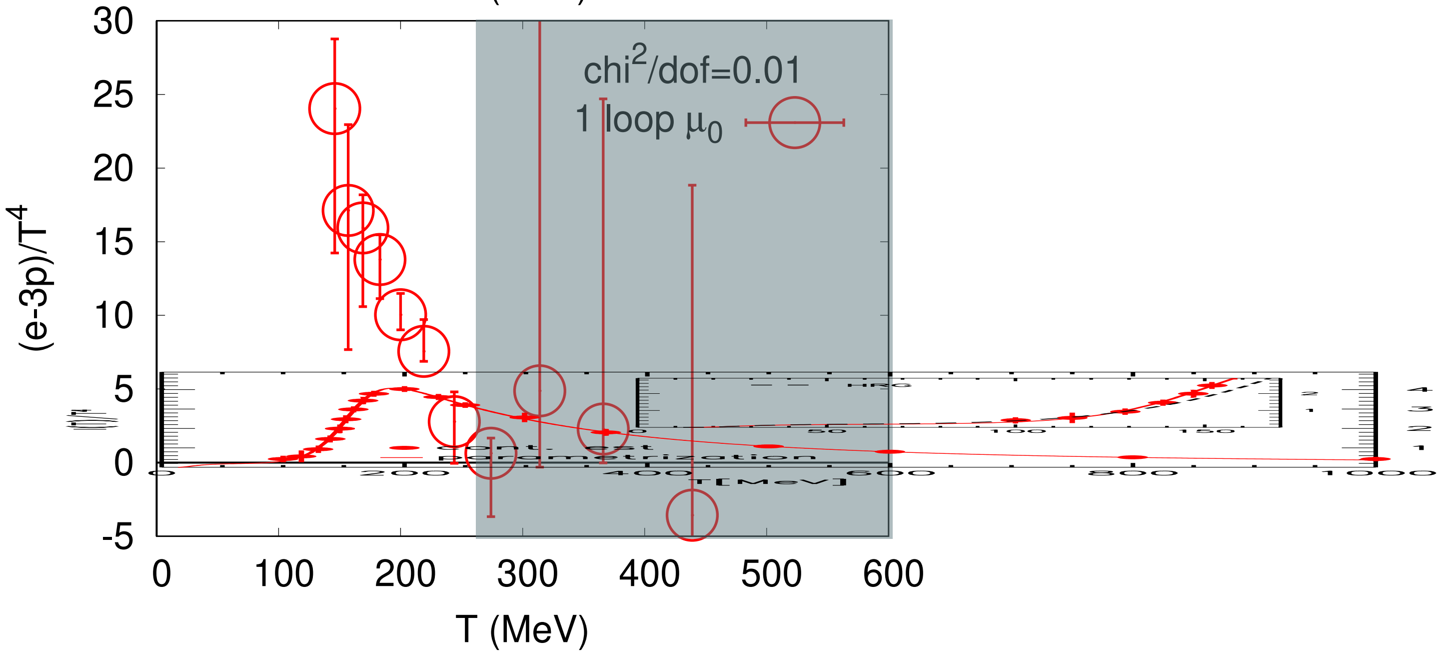
- ☑ μ_0 and μ_d results consistent with each other
- ☑ μ_0 improves linear behavior at large t

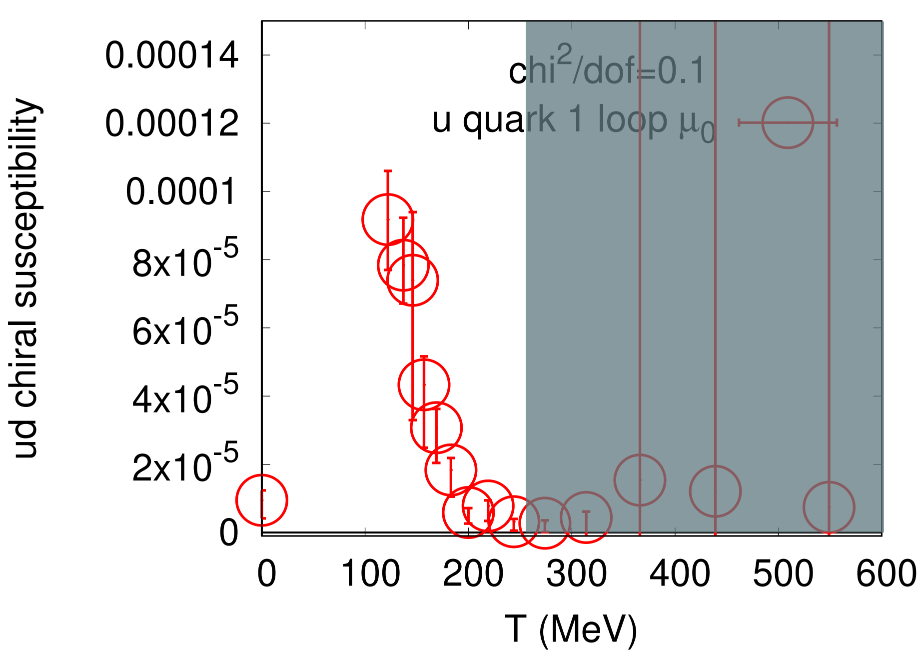
=> μ_0 extend the reliability/applicability of the SF*t*X method

Results with the μ_0-scale



Borsany et al., JHEP 1011, 077 (2010), with KS(stout).





- ☑ **T_{pc}^{phys} < 157 MeV (*T*≈122-146MeV critical ??)**

  (cf.) Result with 2+1 staggered quarks
  156.5 ± 1.5 MeV
  Bazavov et al. PLB795, 15 (2019), HISQ

- ☐ Need more statistics / more data points at low *T*'s. (*on-going*)
- ☐ A definite conclusion possible only after continuum extrapolation.


summary

summary: SF*t*X method in 2+1 flavor QCD

1. 2+1 flavor QCD with slightly heavy u,d and ≈physical s quarks

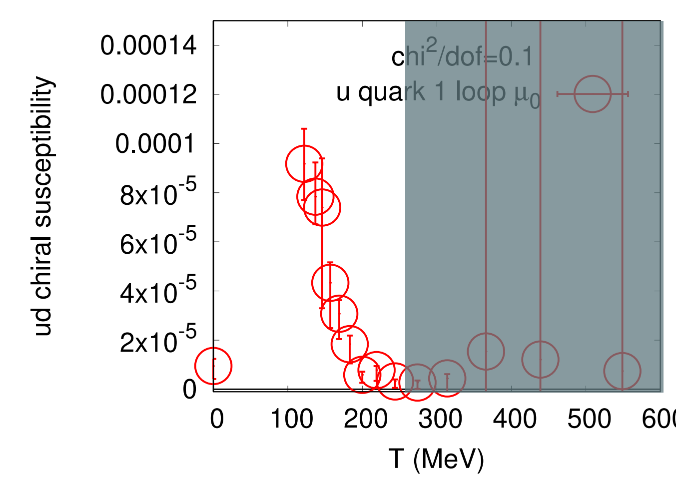
- fine $a \approx 0.07$fm lattice with improved Wilson quarks, $32^3 \times N_t$ (N_t=4-16): $T \approx$ 174-697MeV
  - ☑ EoS agrees well with conventional integral method at $T \leq 300$ MeV ($N_t \geq 10$), while $O((aT)^2 = 1/N_t^2)$ lattice artifacts suggested at $N_t \leq 8$.
  - ☑ Chiral suscept. show clear peak at $T_{pc} \approx 190$ MeV expected from Polyakov loop etc.
  - ☑ Topological suscepts. by gluonic and fermionic definitions agree well.
  - ☑ μ_0-scale extends the reliability/applicability of the SF*t*X method.
  - ☑ 1- and 2-loop matching coefficients lead to consistent results, while EoM gets $O((aT)^2 = 1/N_t^2)$ lattice artifacts at $N_t \leq 10$.

=> SF*t*X powerful in evaluating physical observables.

- ☐ A definite conclusion possible only after continuum extrapolation, though our results suggest that $a \approx 0.07$fm is fine enough.

2. 2+1 flavor QCD with physical u,d,s quarks

- less fine $a \approx 0.09$fm lattice, $32^3 \times N_t$ (N_t=4-18): $T \approx$ 122-549MeV
  - ☑ The μ_0-scale helps much.
  - ☑ **$T_{pc}^{phys} < 157$ MeV ($T \approx$ 122-146MeV critical ??)**
- ☐ Need more statistics / more data points at low T's. => *on-going.*
- ☐ Data at larger t/a^2 may help. => *on-going.*
- ☐ Need continuum extrapolation too. => *being started.*

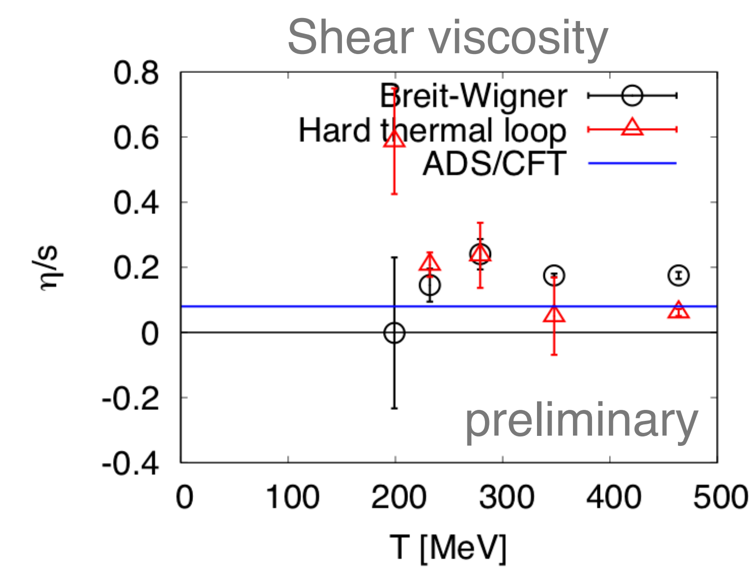


preliminary

prospects / to do

☆ other observables

- EMT correlation functions
  - transport coefficients of QGP: shear/bulk viscosity, etc.
  - test: thermodynamic relations vs. linear response relations
- chiral observables
  - matrix elements: B_K, etc.
- topological observables at the physical point



☆ continuum extrapolation

- Slightly heavy ud + ≈phys. s on a **less fine lattice** (a≈0.097fm), 24^3xN_t (N_t=8-12): T≈170-254MeV
  - ☑ Look similar to the fine lattice case
  - ☑ Linear windows narrower than the fine lattice case. => μ_0 will help
  - ☑ *a*-dep. looks small up to this *a*
  - ☐ need more statistics + a finer point

- PACS10 configurations (T=0) at the physical point
  - ☐ generation of finite temperature configurations => started!