Numerical simulation of superstring theory at finite temperatures

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Mainly based on collaborations with
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1. Introduction

Elementary particles and String Theory

- The Standard Model of particle physics cannot consistently include gravitons.

→ Superstring Theory can!

- "Unification" by Superstring Theory
  - ☑ Graviton ⋯ closed strings
  - ☑ Gauge bosons
  - ☑ Leptons and Quarks
  - ☑ Higgs boson

  } open strings + D-branes

String theory describes
Interactions, Matters and Space-time
in a unified way

1. Introduction

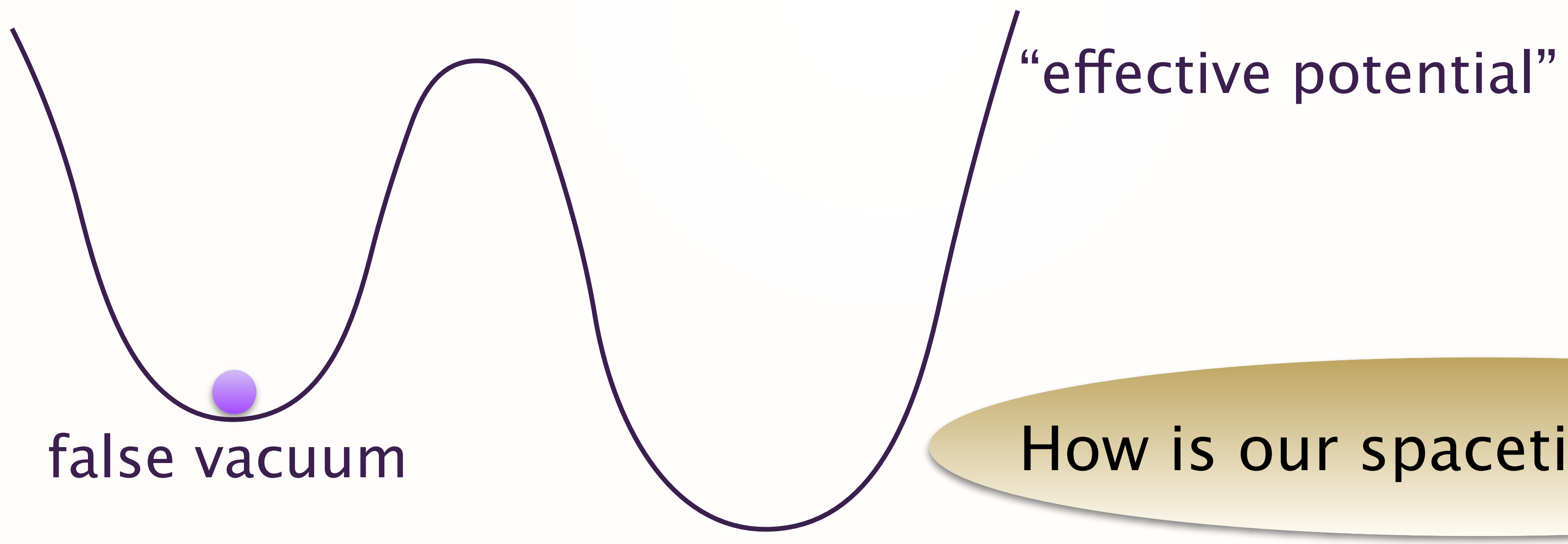
Problem in String Theory

- Spacetime dim. is restricted to 10D or 11D.

  ⋯ But we want 4D spacetime

→ Solved if extra dimensions are compactified in the ground state

Problem:
String theory, based on perturbation theory, has almost infinitely many candidates of the ground state (i.e. the true vacuum).



How is our spacetime realised?

We need non-perturbative formulation

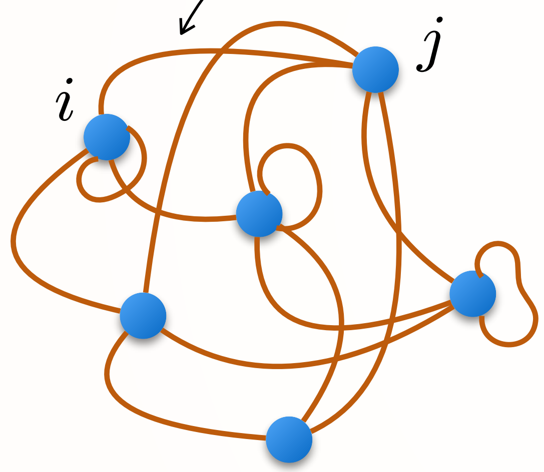
1. Introduction

Non-perturbative formulations of String Theory

String Field Theory, Matrix Model, AdS/CFT duality, …

We discuss formulations given by matrices in this talk

One way to understand such matrix formulations is a brane picture:



$$X = \begin{pmatrix} X_{11} & & X_{ij} \\ & \ddots & \\ & & X_{NN} \end{pmatrix}$$

branes (soliton-like objects)
connected by strings

The idea is to describe quantum gravity by a gauge theory.

SU(N) Gauge Theory ⟷ **String Theory with gravity**

The formulations have been intensively and extensively tested.

1. Introduction

Formulations by matrices

- Described by a lower dimensional gauge theory
  - IKKT model (0 dim.) [Ishibashi, Kawai, Kitazawa, Tsuchiya '96]
  - BFSS model (1 dim.) [de Wit, Hoppe, Nicolai '88; Banks, Fischler, Shenker, Susskind '96]
  - $\mathcal{N}=4$ Super Yang-Mills (3+1 dim.) [Maldacena '97]

- The entire spacetime is not a priori.

  → It is expected to emerge in the large-N limit.

- (1st principle) Numerical computation
  - Emergent spacetime and expanding universe in Matrix Model

    [Kim, Nishimura, Tsuchiya '11;
    Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis '20; ...]

  - Numerical tests of AdS/CFT duality

    [Anagnostopoulos, Hanada, Nishimura, Takeuchi '07;
    Catterall, Wiseman '07; Hanada, Hyakutake, Ishiki, Nishimura '13; ...]

1. Introduction

Emergence of spacetime and branes

- How does spacetime emerge in matrix models?
- In string theory, there are soliton-like objects—branes. How do they realise in matrix models?

→ The BMN model partially answered these questions.

BMN model

[Berenstein, Maldacena, Nastase '02]

- Considered to be non-perturbative formulation of M-theory
- 1D super quantum mechanics with $SU(N)$ gauge sym.

$$S = N\int dt\,\mathrm{Tr}\Big[\frac{1}{2}(D_t X^a)^2 + \frac{1}{2}(D_t X^m)^2 - \frac{1}{4}\Big(\frac{\mu}{3}\epsilon_{abc}X^c - i[X^a, X^b]\Big)^2 + \frac{1}{2}[X^a, X^n]^2 + \frac{1}{4}[X^m, X^n]^2 - \frac{\mu^2}{72}X^m X^m + \text{fermions}\Big]$$

$a,b = 1,2,3, \quad m,n = 4,\dots,9$

Symmetry: $R \times SO(3) \times SO(6)$

X^a, X^m and 16 fermions are $N\times N$ matrices

mass parameter

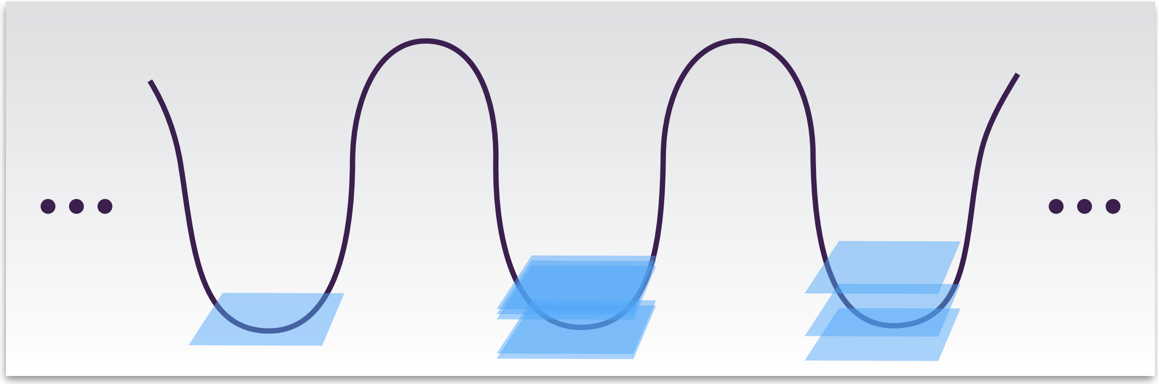
1. Introduction

Vacua and Branes

- Branes are realised typically as classical configurations, in matrix models.

- The BMN model has many discrete vacua degenerated.

[Kim, Plefka '02; Dasgupta, Sheikh-Jabbari, Raamsdonk '02]



Each vacuum corresponds to a brane configuration.

[Maldacena, Sheikh-Jabbari, Raamsdonk '02; Lin, Lunin, Maldacena '04]

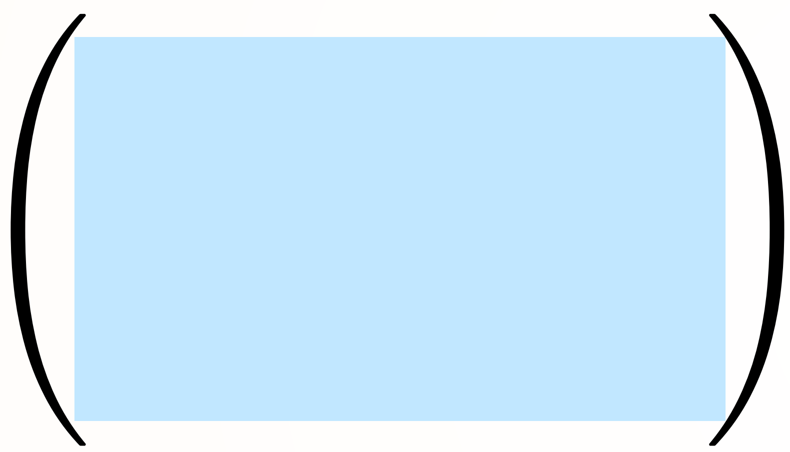
- The brane geometry can be reproduced around a vacuum by matrix eigenvalues.

[Y.A., Ishiki, Shimasaki, Okada '14; Y.A., Ishiki, Shimasaki, Terashima '17]

How do they dynamically realise?

2. What to Expect

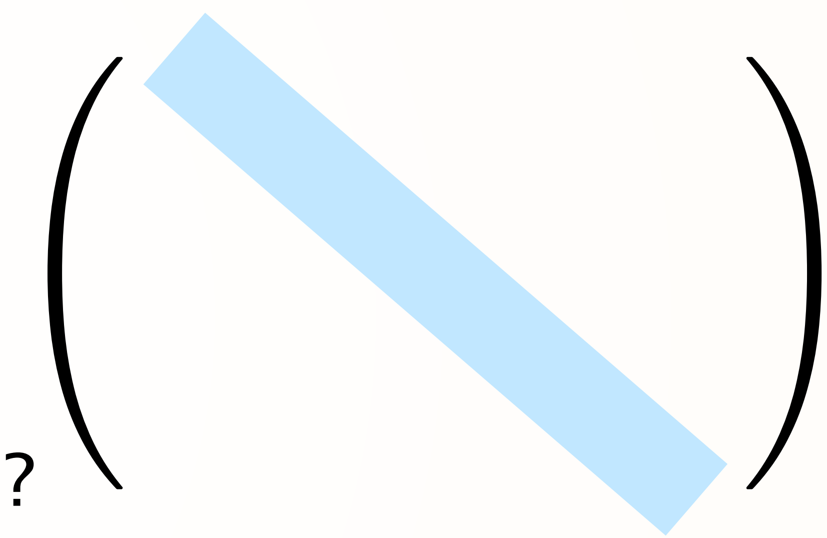
How does the geometry emerge
when we scale the Temperature?

$X_i =$ 

just matrices

high energy scale

phase transition?



low energy scale

※ The vacua are nearly diagonal
(so are geometrical matrices in general)

2. What to Expect

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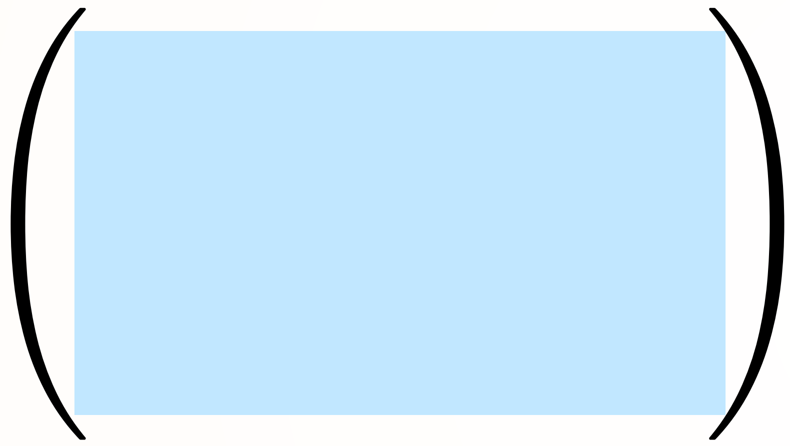
$X_i =$ 

phase transition?

just matrices → geometry

high energy scale — low energy scale

※ The vacua are nearly diagonal
(so are geometrical matrices in general)

2. What to Expect

Confinement/deconfinement transition

- The gauge field A has holonomy at finite temperatures.

  Polyakov loop: $P = \dfrac{1}{N}\,\mathrm{Tr}\, e^{i\oint d\tau A}$ (τ: imaginary time)

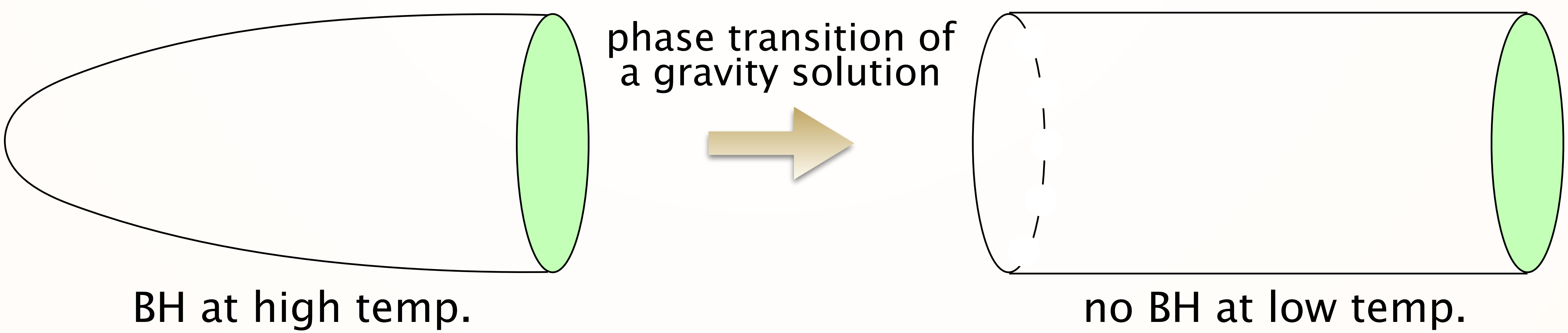
  This is an order parameter that detects whether dominant states behave such that the colour d.o.f. are confined or "visible."

  $\langle P \rangle = 0$ $\cdots$ confined phase (at low temp.)

  $\langle P \rangle \neq 0$ $\cdots$ deconfined phase (at high temp.)

- It corresponds to black-hole transition on the gravity side.

[Costa, Greenspan, Penedones, Santos '14]

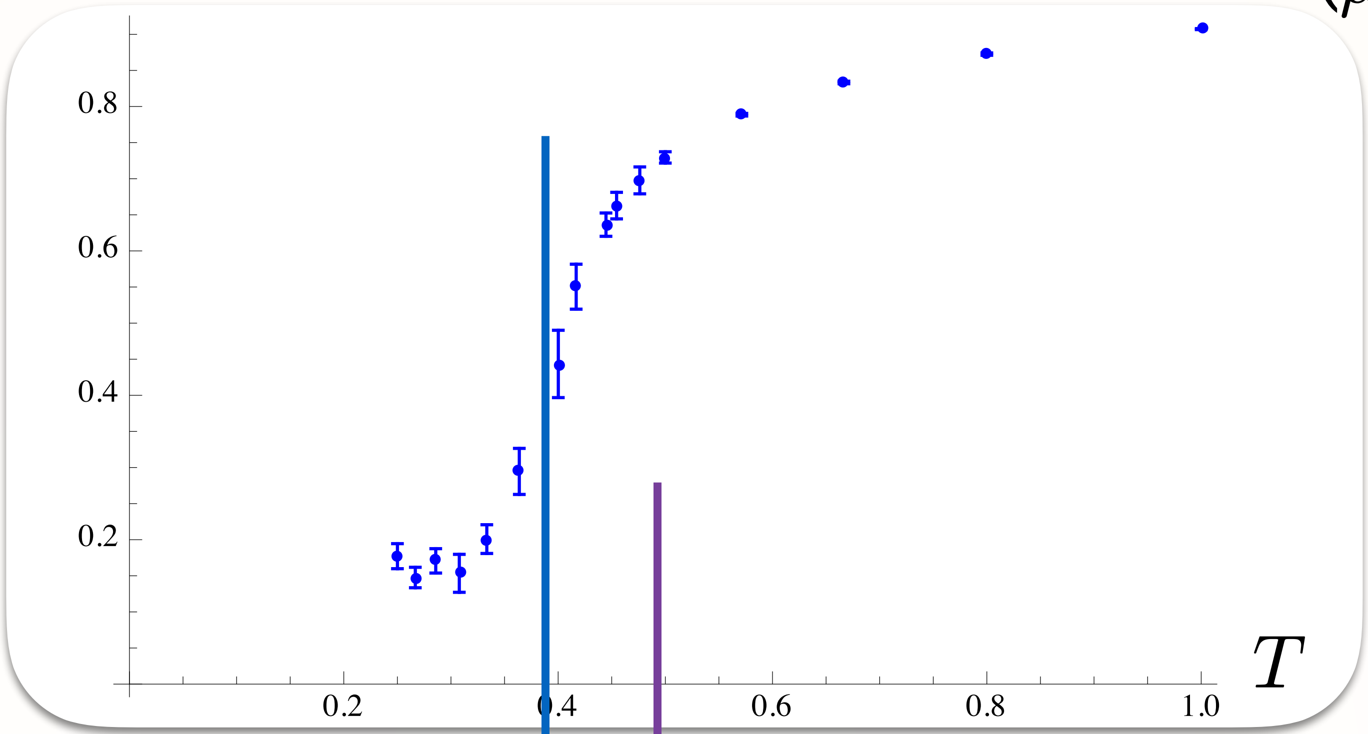


3. Lattice Simulation

$(\mu=5,\ \Lambda=24,\ N=11)$

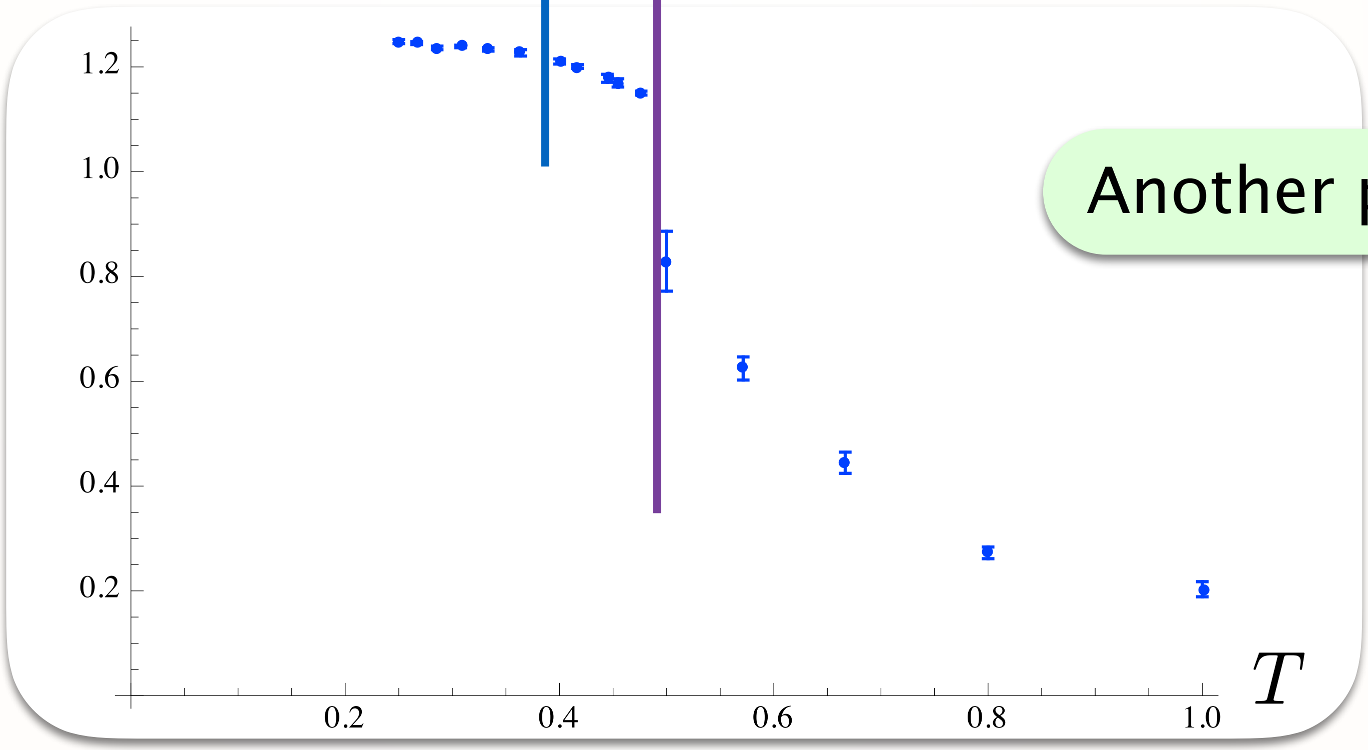
Polyakov loop:

$\langle |P| \rangle$



Myers term:

$\sim \langle \mathrm{Tr}\,(iX_1[X_2, X_3]) \rangle$

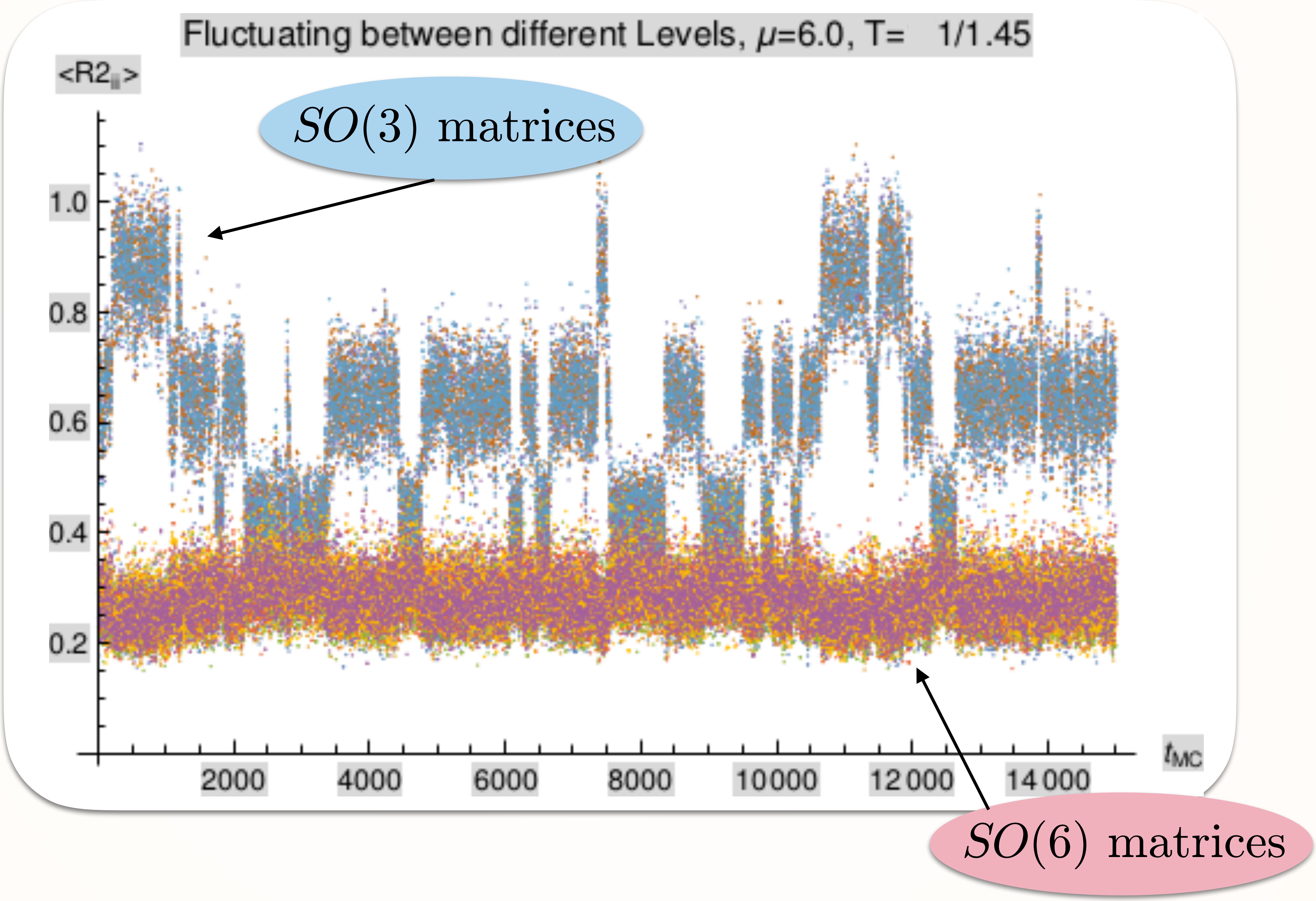


Another phase transition!

[Y.A., Filev, Kovacik, O'Connor '18]

3. Lattice Simulation

$\sim \mathrm{Tr}[X_i X_i]/N$ $(\mu{=}6,\ \beta{=}1.45,\ \Lambda{=}24,\ N{=}8)$



[Y.A., Filev, Kovacik, O'Connor '18]

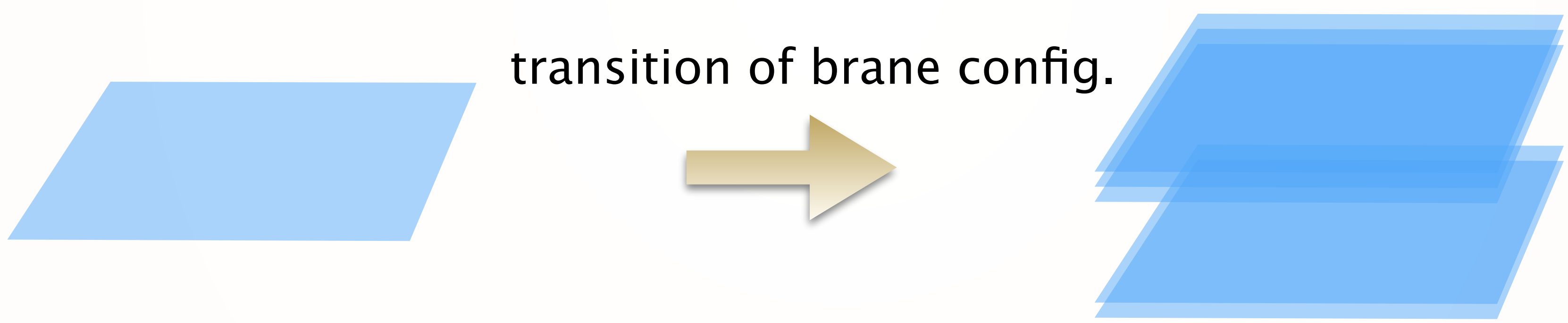
3. Lattice Simulation

Myers transition

- Phase transitions between different vacua of the matrix model were observed for the first time.

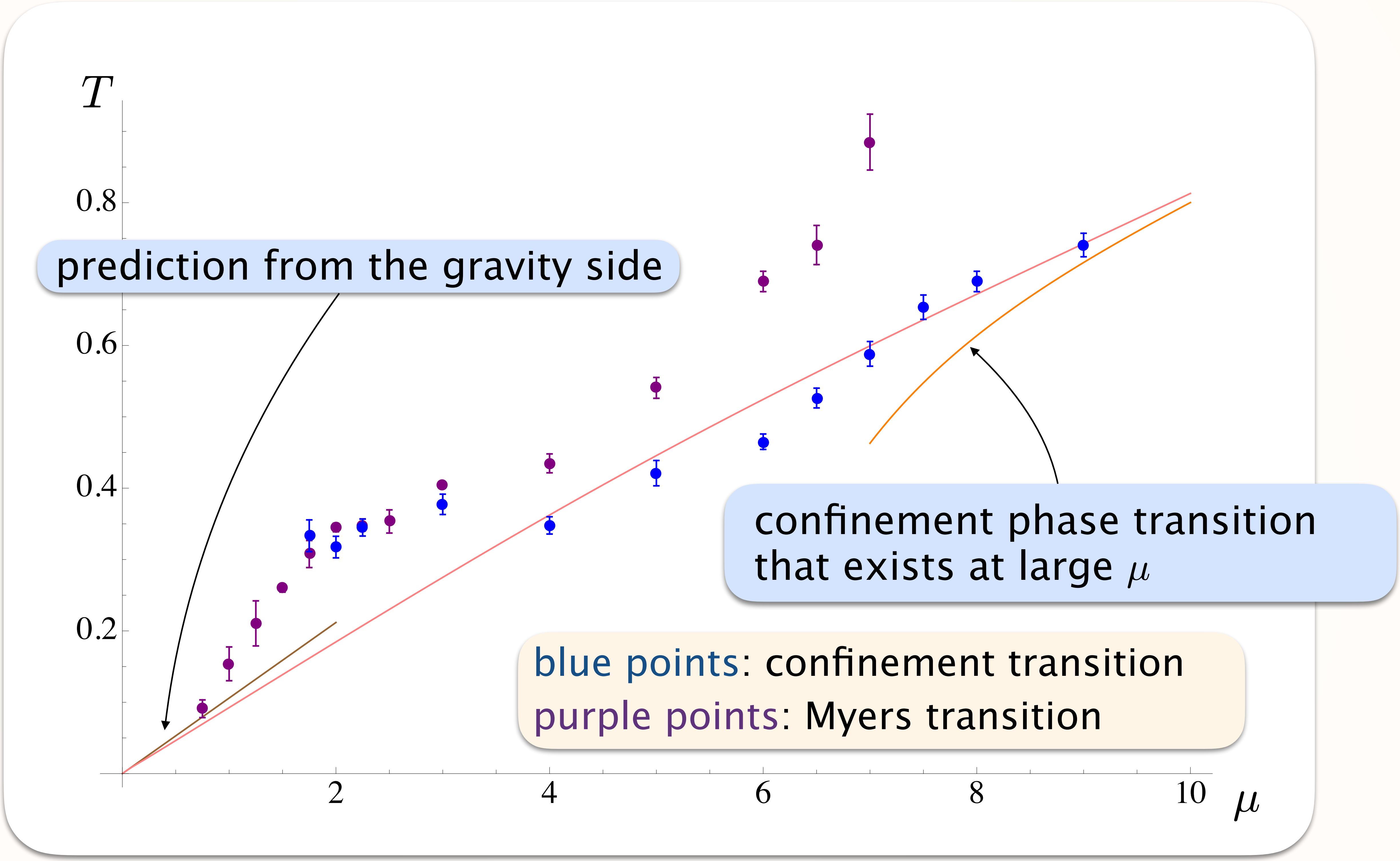
⋯ transitions b/w different brane configurations

i.e. transitions b/w spacetimes generated by different config.



3. Lattice Simulation

(Λ=24, N=8)



The simulation results AGREE with theoretical predictions.

[Y.A., Filev, Kovacik, O'Connor '18]

4. Summary

- Observed two phase transitions:
  confinement transition and Myers transition
  They are consistent with the gravity prediction so far, and don't merge at least on the lattice with finite N at $3 \lesssim \mu \lesssim 6$.
- The Myers transition looks like "no geometry → geometry".
- Since there are many discrete vacua in the matrix model, we expect a richer structure at lower temperatures.

Bosonic BMN model

- We found there are 2 would-be transitions at finite N, which merge into 1 confinement transition in the large-N limit.
- Finite-N effect in the transition is reasonably understood.
- No Myers transition

[Y.A., Kovacik, O'Connor '20, and to appear]

Longitudinal 5-branes

- Adding another type of brane is also interesting. A $\mu=0$ version was numerically studied.

[Filev, O'Connor '15; Y.A., Filev, Kovacik, O'Connor '16]