Femtoscopic study in high-energy heavy-ion collisions

TCHoU Research Member Meeting
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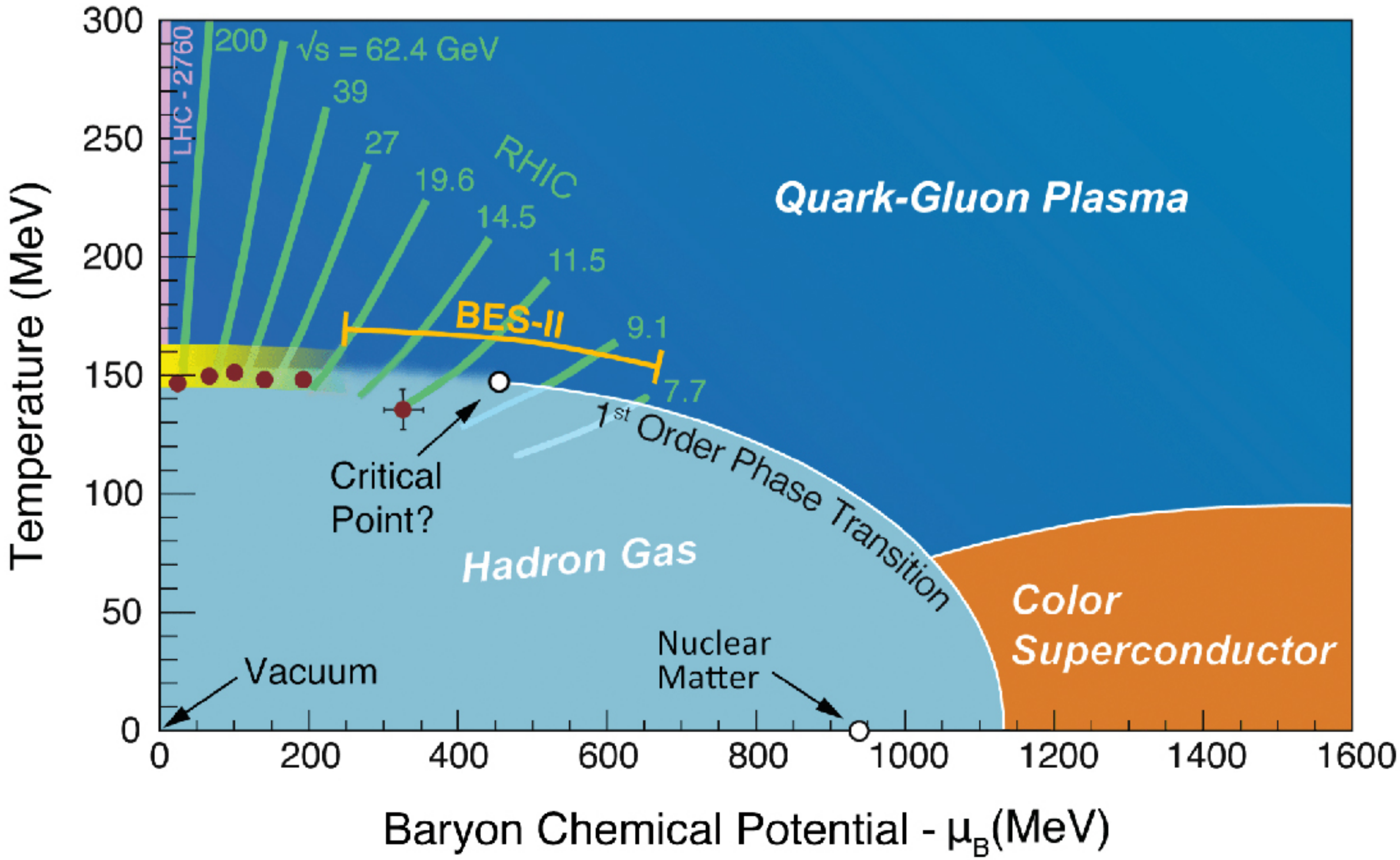
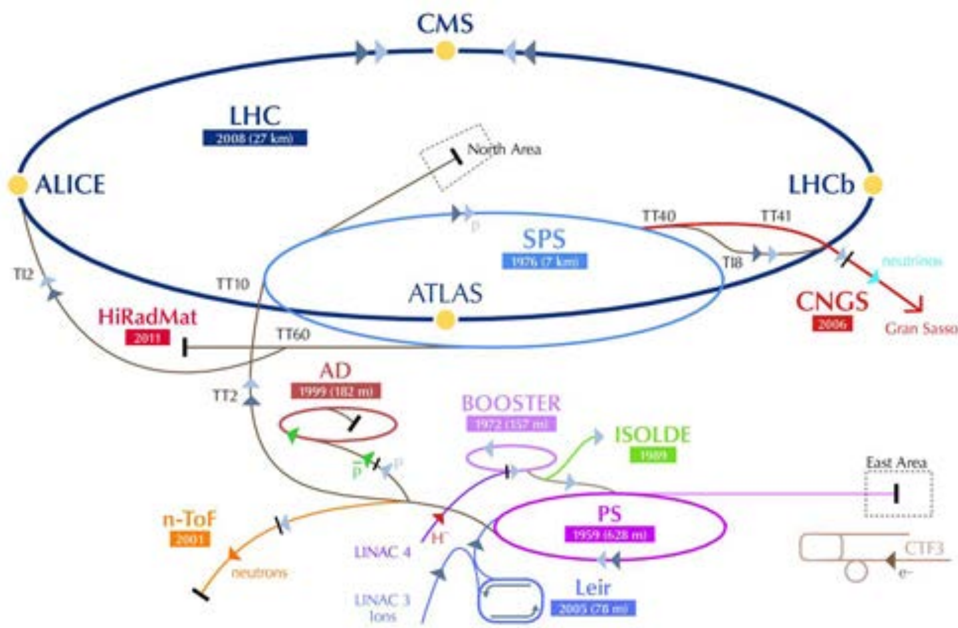
Introduction

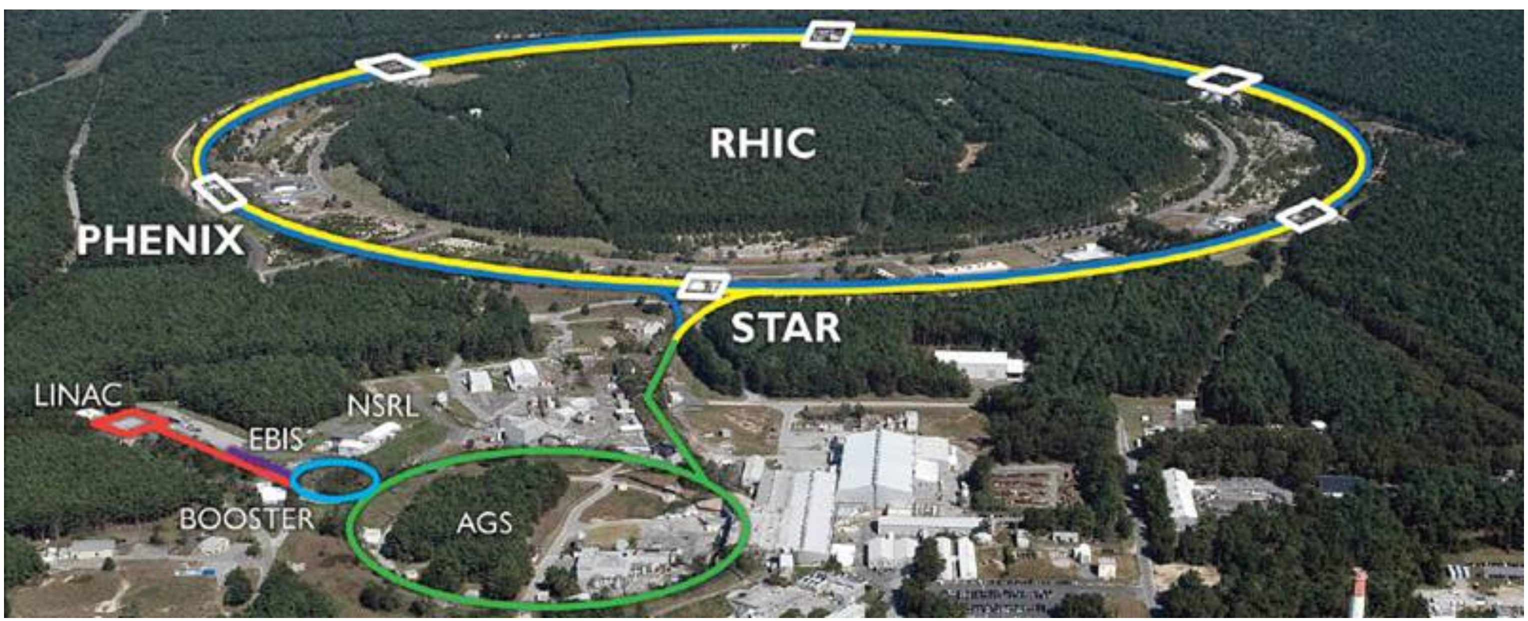


- Lattice QCD predicts QGP at ~150 MeV temperature
- Big-Bang theory of the universe predicts that matter existed in QGP form after ~1μs of the universe formation

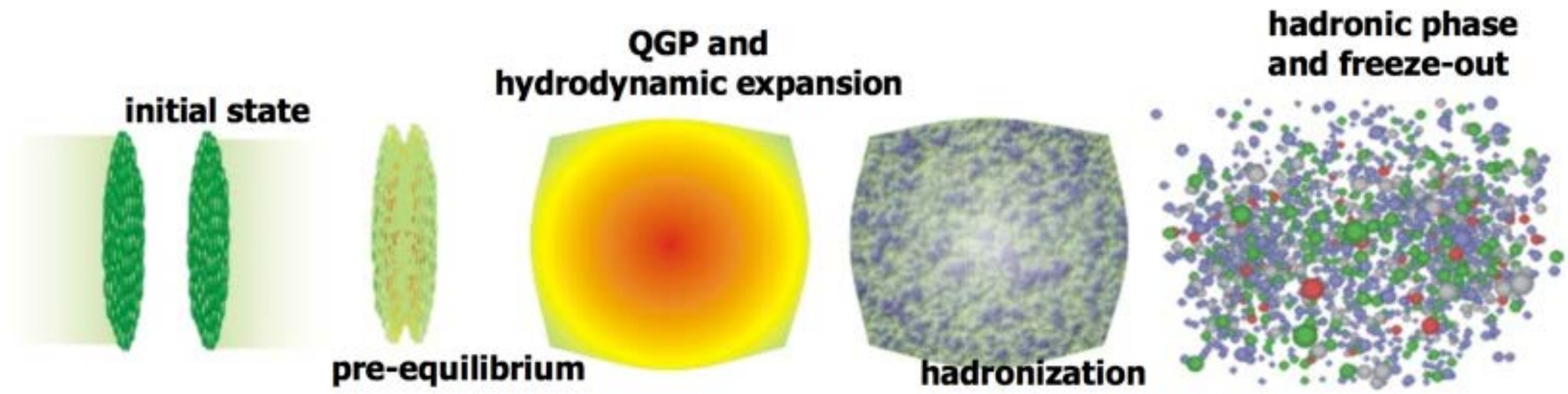
Relativistic Colliders

- Compressed Baryonic Matter (CBM) at FAIR/GSI and J-PARC heavy-ion experimental project at KEK/JAEA





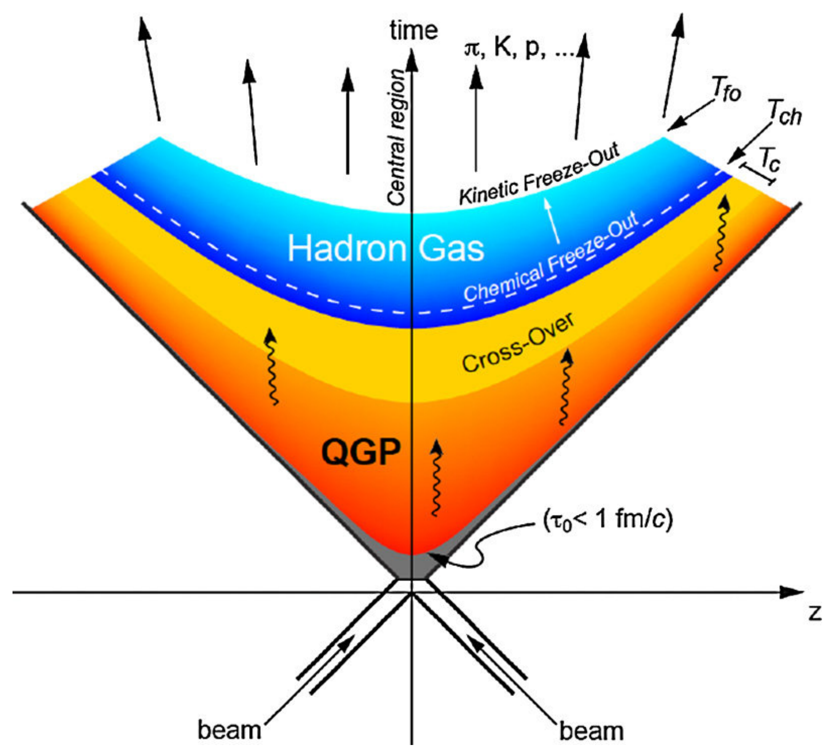
Heavy-ion collisions



- Heavy-ions like Gold (Au^{79+}) and Lead (Pb^{82+}) are allowed to collide at ultra-relativistic speed in relativistic colliders like the LHC and RHIC

- Very hot and dense QGP like medium formation in heavy-ion collisions due to inelastic collisions between nuclei and conversion of kinetic energy into heat

- To obtain the equation of state, it is important to know the dimensions of fireball which is impossible to measure directly due to its very small size (size ~10^{-15} m, lifetime ~ 10^{-23} s)



Introduction (contd.)

- Femtoscopy (or HBT technique) provides a direct tool to measure the source size
- Probe space-time characteristics of the source using particle correlations in momentum space
- Main sources of correlations:

➡ **Quantum statistics (QS)** → **HBT analysis**

- Identical bosons - Bose-Einstein quantum statistics
- Identical fermions - Fermi-Dirac quantum statistics

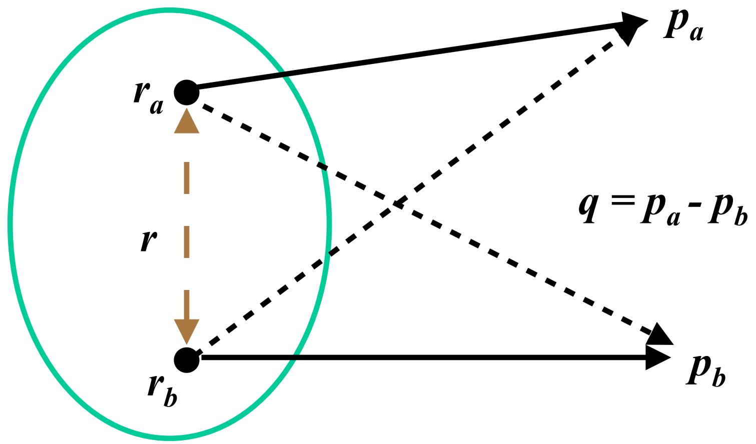
➡ **Final-state interactions (FSI)**

- Strong interaction
- Coulomb interaction (repulsion/attraction)

QS + FSI→ Femtoscopic analysis

Two-particle correlation function

$$C_2 = \frac{P_2(p_a, p_b)}{P_1(p_a)P_1(p_b)}$$



Koonin-Pratt Equation,

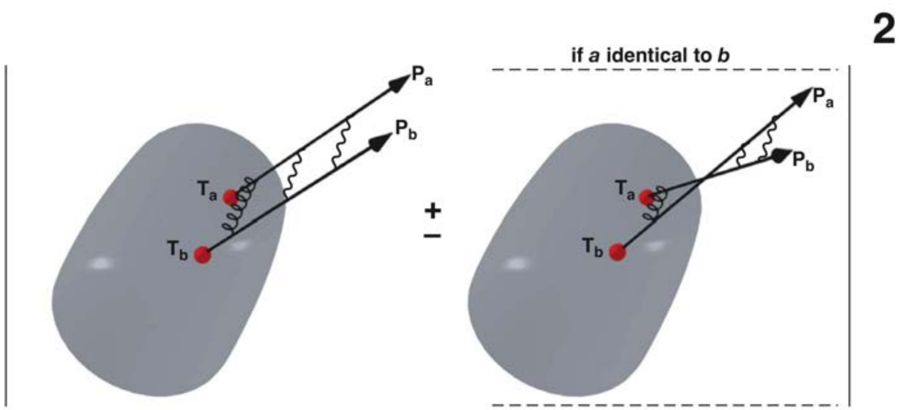
PLB70 (1977) **43**,*S. Pratt et al.*, PRC42 (1990) 2646

Source function

$$\mathbf{C(q)} = \int \mathbf{dr}|\psi(\mathbf{q},\mathbf{r})|^2\mathbf{S(r)}$$

Experimental correlation function

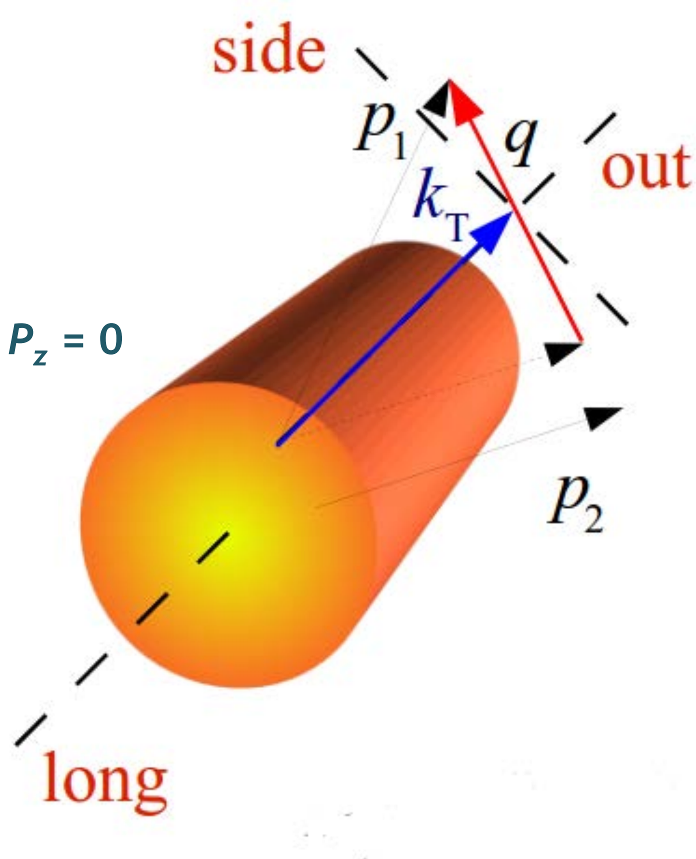
Pair wave function



- $P_2(p_a,p_b)$ – probability of detection of particles with momenta p_a and p_b
- $P_1(p_i)$ – probability of detection of particle with momentum p_i

Bertsch–Pratt 'Out-Side-Long' coordinate system

- **side** is interpreted as **geometric size**
- **out** gives info on **emission process**
- **long** is used for **emission time approx.**



Longitudinal co-moving system (LCMS):

A rest frame moving along the beam direction such that $P_z = 0$

$$V_{long} = (P_0 V_z - P_z V_0)/M_T$$

$$V_{side} = (P_x V_y - P_y V_x)/P_T$$

$$V_{out} = (P_x V_x + P_y V_y)/P_T$$

Pair rest frame (PRF):

$$V'_{out} = \frac{M_{inv}}{M_T} \frac{(P_x V_x + P_y V_y)}{P_T} - \frac{P_T}{M_T M_{inv}} P.V$$

Where $M_T{}^2 = P_0^2 - P_z^2$, $P_T^2 = P_x^2 + P_y^2$, $M_{inv}^2 = P^2$ and $k_T = (p_{T1} + p_{T2})/2$

Experimental Correlation function

$$\mathbf{C(q) = \frac{S(q)}{B(q)}}$$

- **S(q) -** distribution of q of pairs from same events (signal)
- B**(q) -** distribution of q of pairs from different events (background)

where q : relative momentum of the pair particles

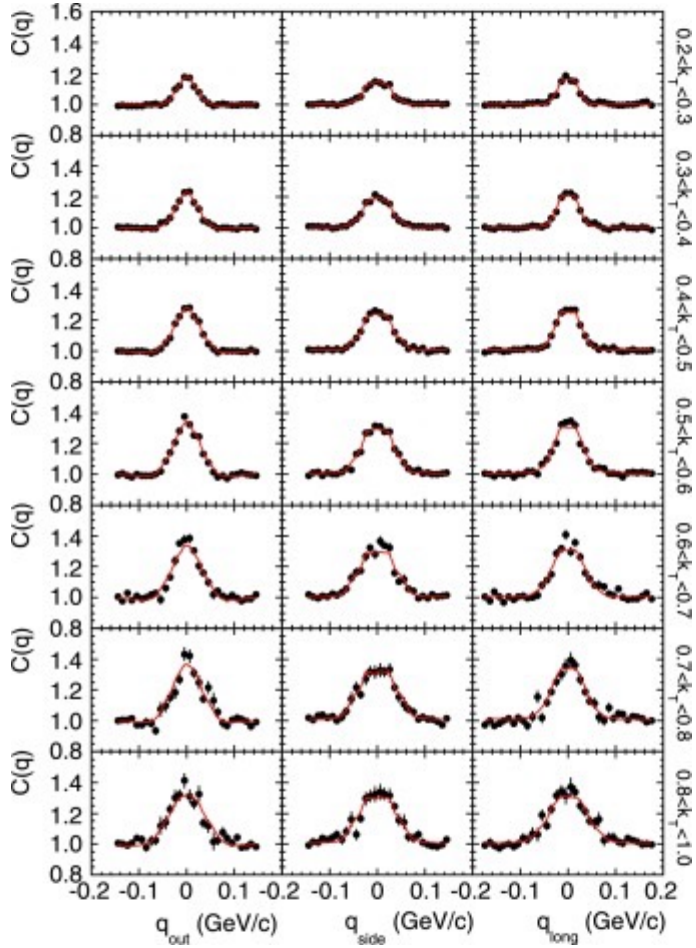
Fit function for identical charged pions and kaons:

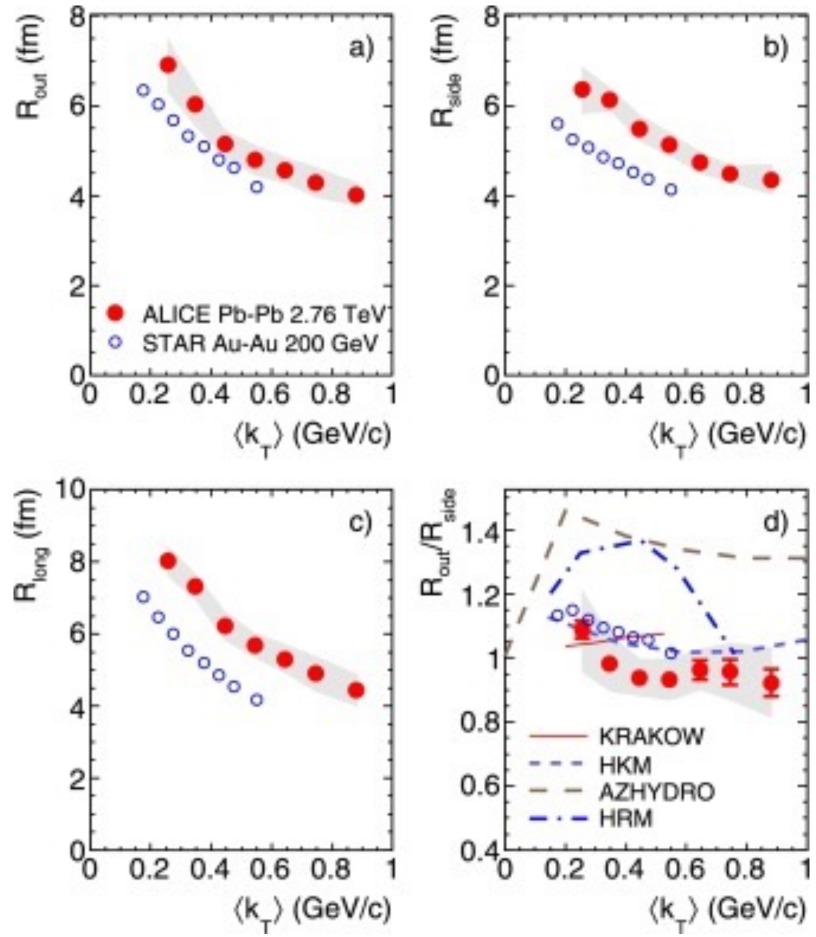
$$C(q) = N[(1-\lambda) + \lambda K(q_{inv})(1+G(q))]$$

$$G(q) = exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2)$$

★ λ - fraction of the pairs originate from the spatio-temporal region relevant for correlations,
★K(q_{inv}) - squared Coulomb wave function averaged over a spherical source

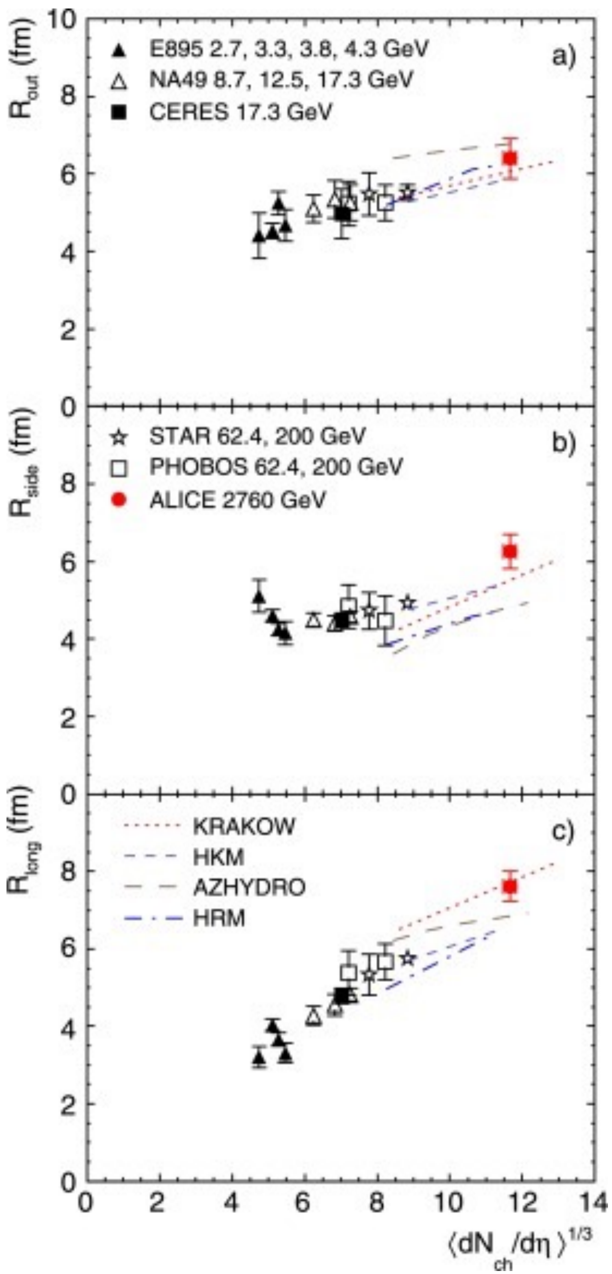
Identical pion-pion femtoscopy

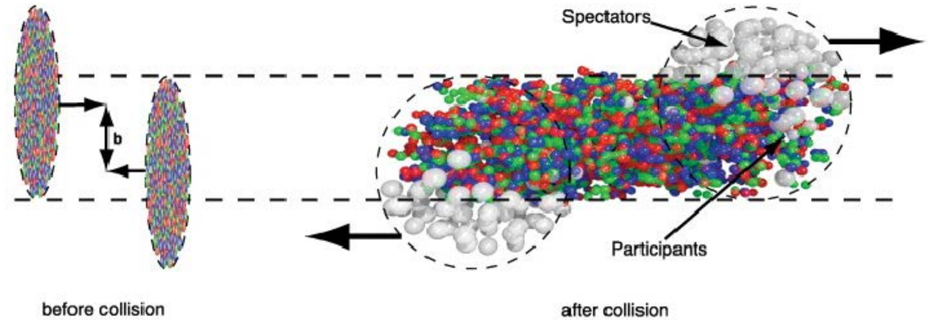


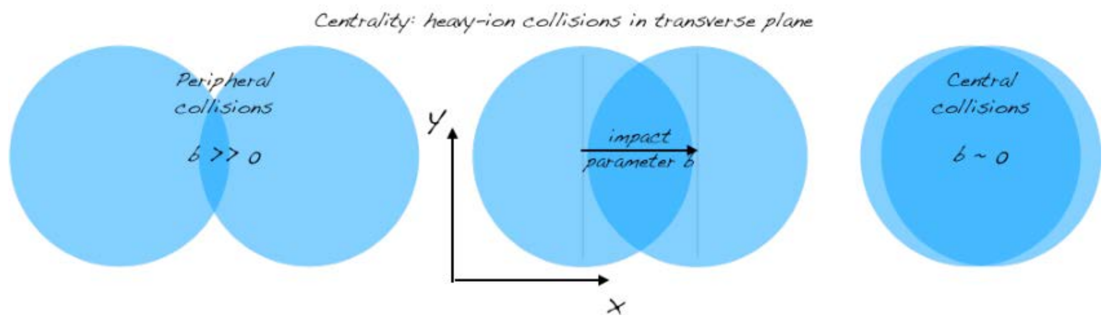


- Source radii decreases with increasing average transverse momentum

Identical pion-pion femtoscopy (cont.)







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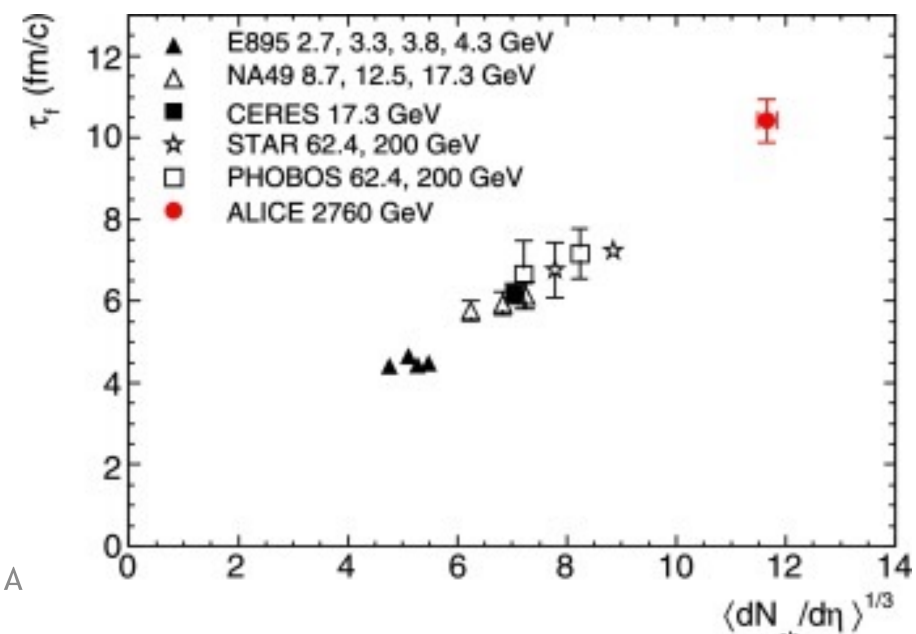
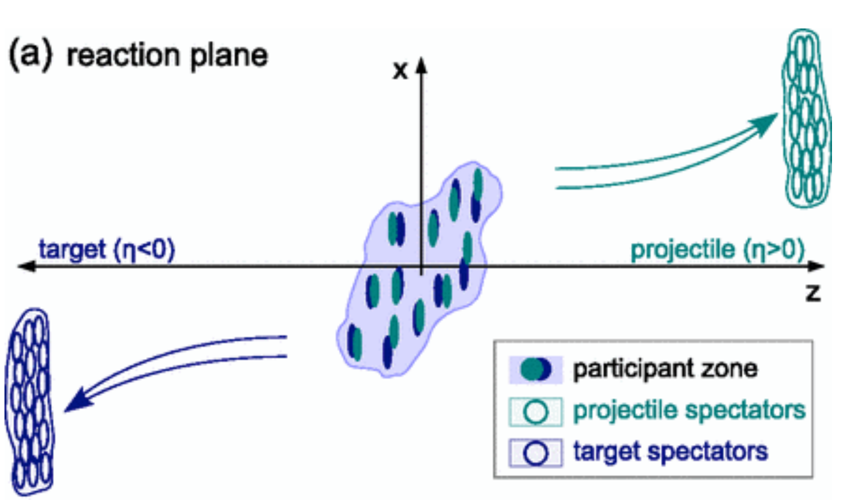
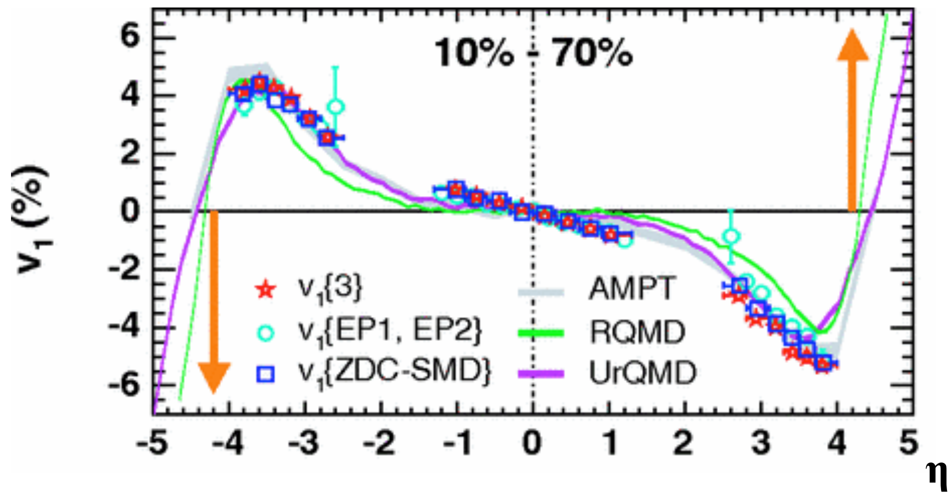


Femtoscopic studies w.r.t. Ψ_1

ALICE Collaboration, Phys. Rev. Lett. 111 (2013) 232302



STAR Collaboration, Phys. Rev. C 73 (2006) 34903

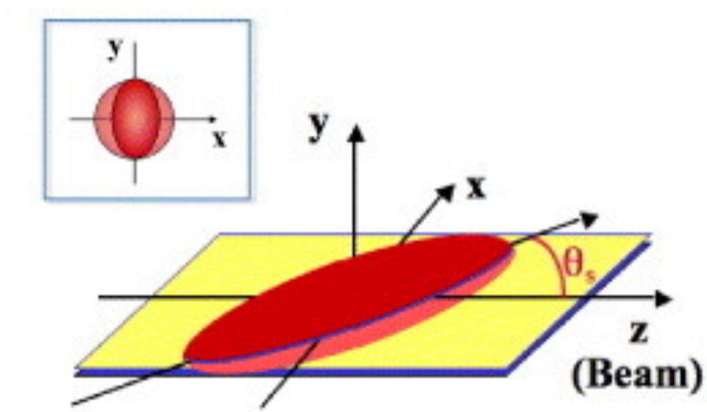


Au + Au 62.4 GeV, charged particle v_1

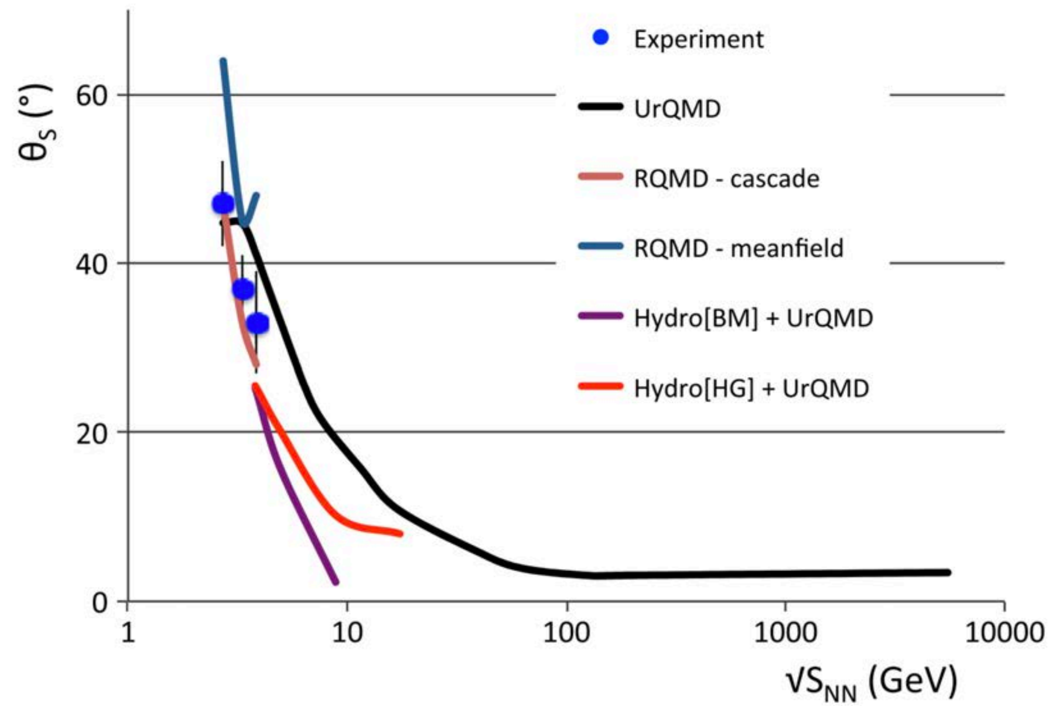
- Directed flow v_1 (= $\langle\cos(\phi - \Psi_1)\rangle$) is produced due to interaction between spectator and participant particles
- $v_1(\eta)$ is zero three times at around mid, forward and backward rapidities : possible signature of phase transition (J. Brachmann et al. Phys. Rev. C 61 (2000) 024909)
- v_1 can't be explained by hydrodynamical models unlike v_2 or v_3

Femtoscopic studies w.r.t. Ψ_1

- v_1 signal can be generated from assuming the "tilted source" initial conditions
- Femtoscopic measurements w.r.t. Ψ_1 can give the information about tilt angle θ_s



M. A. Lisa et al. New J. Phys. 13 (2011) 065006



Femtoscopic studies w.r.t. Ψ_1

- Fit function for identical particles with cross term:

$$C(q) = N[(1-\lambda) + \lambda K(q_{inv})(1+G(q))]$$

$$G(q) = exp(-R_{out}^2 q_{out}^2 - R_{side}^2 q_{side}^2 - R_{long}^2 q_{long}^2 - R_{os}^2 q_{out} q_{side} - R_{ol}^2 q_{out} q_{long} - R_{sl}^2 q_{side} q_{long})$$

Fit function

$$\begin{cases} R_{\mu,0}^2 + 2R_{\mu,1}^2 cos(\phi - \Psi_1) + 2R_{\mu,2}^2 cos(2(\phi - \Psi_1)), (\mu = o, s, l, ol) \\ R_{\mu,0}^2 + 2R_{\mu,1}^2 sin(\phi - \Psi_1) + 2R_{\mu,2}^2 sin(2(\phi - \Psi_1)), (\mu = os, sl) \end{cases}$$

$$\theta_s = \frac{1}{2}\tan^{-1}\left(\frac{-4R_{sl,1}^2}{R_{l,0}^2 - R_{s,0}^2 + 2R_{s,2}^2}\right)$$

Event plane reconstruction:

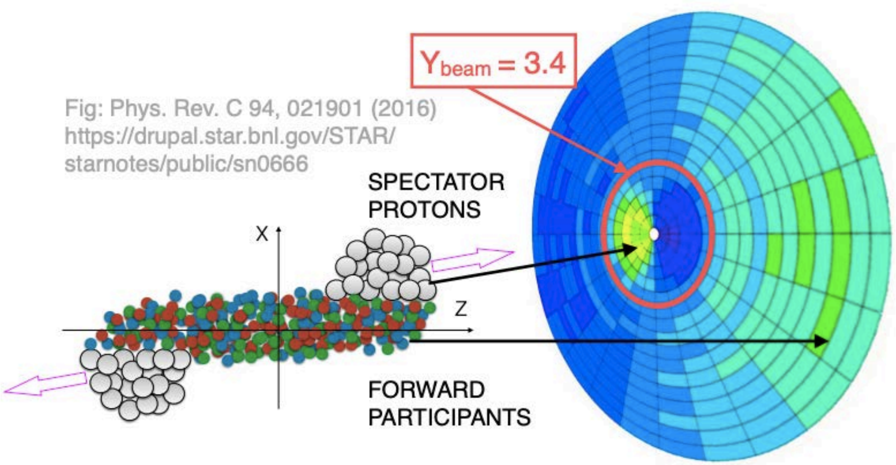
$$\Psi_n = \frac{1}{n}\tan^{-1}\left(\frac{Q_{x,n}}{Q_{y,n}}\right) = \frac{1}{n}\left(\frac{\sum w_i \sin(\phi_i)/\sum w_i}{\sum w_i \cos(\phi_i)/\sum w_i}\right)$$

- $w_i = \eta$ for TPC
- **w_i (For EPD) = pm*wgr[ringID]*aa**
- ringID = Tile/2 ,
- double wgr[16] = { 6.5, 4.0, 2.5, 1.0, 0.0, 0.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0, -1.0};
- Float_t pm =1; if (Id<0) pm=-1;
- aa= NMip ;if (NMip<0.3) aa=0.; else if (NMip>3.0) aa=3.0;

Recentering:

$$Q'_{x,n} = \frac{Q_{x,n} - \langle Q_{x,n}\rangle}{\sigma_{Q_{x,n}}}$$

$$Q'_{y,n} = \frac{Q_{y,n} - \langle Q_{y,n}\rangle}{\sigma_{Q_{y,n}}}$$



Flattening:

$$\Psi''_n = \Psi'_n + \sum_{k=1} \frac{2}{k}\left\{-\langle\sin(kn\Psi'_n)\rangle\cos(kn\Psi'_n) + \langle\cos(kn\Psi'_n)\rangle\sin(kn\Psi'_n)\right\}$$

➡ k is taken upto 8

Event plane resolution:

$$\mathrm{Res}\{\Psi_{n,a}\}\mathrm{Res}\{\Psi_{n,b}\} = \langle cos(n[\Psi_{n,a} - \Psi_{n,b}]\rangle$$

$$\mathrm{Res}\{\Psi_{n,b}\}\mathrm{Res}\{\Psi_{n,c}\} = \langle cos(n[\Psi_{n,b} - \Psi_{n,c}]\rangle$$

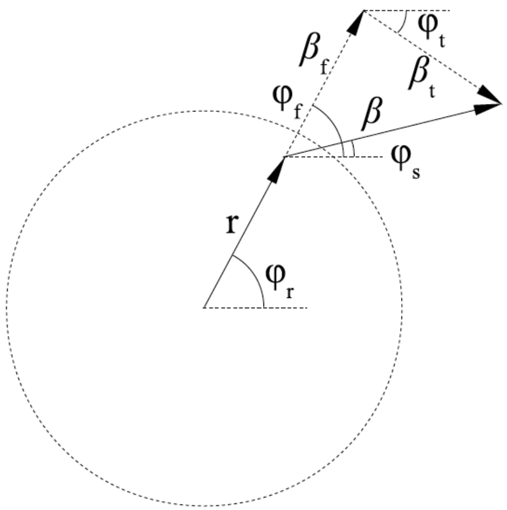
$$\mathrm{Res}\{\Psi_{n,c}\}\mathrm{Res}\{\Psi_{n,a}\} = \langle cos(n[\Psi_{n,c} - \Psi_{n,a}]\rangle$$

$$\mathrm{Res}\{\Psi_{n,a}\} = \sqrt{\frac{\langle cos(n[\Psi_{n,a} - \Psi_{n,b}]\rangle\langle cos(n[\Psi_{n,c} - \Psi_{n,a}]\rangle}{\langle cos(n[\Psi_{n,b} - \Psi_{n,c}]\rangle}}$$

➡ TPC - detector a, EPD East - detector b, EPD West - detector c

$$\mathrm{Res}\{\Psi_{n,EPD}\} = \langle cos(n[\Psi_{n,EPD} - \Psi_{n,TPC}]\rangle/\mathrm{Res}\{\Psi_{n,TPC}\}$$

Non-identical particles femtoscopy



Adam Kisiel, *Phy.Rev.C* **81**, 064906 (2010)

- In a hydrodynamical induced system :

$$\beta_{particle} = \beta_f + \beta_t$$

component of mean emission point of
a single particle parallel to the velocity

$$\langle x_{out} \rangle = \frac{\langle r\beta_f \rangle}{\left\langle \sqrt{\beta_t^2 + \beta_f^2} \right\rangle} = \frac{r_0 \beta_0 \beta}{\beta_0^2 + T/m_t}$$

assume a Gaussian density profile with radius r_0 and linear transverse velocity profile $\beta_f = \beta_0 r / r_0$ then we

emission asymmetry

$$\mu_{out}^{light,heavy} = \left\langle r_{out}^{light,heavy} \right\rangle = \left\langle x_{out}^{light} - x_{out}^{heavy} \right\rangle$$

- Lighter particles emitted closer to the centre/later than heavier particles
- Emission asymmetry only arises in a system where both random (thermal) and correlated (flow) velocities exist and are comparable in magnitude

Extracting the source size and emission asymmetry

$$\mathbf{C}(\mathbf{k}^*) = \int \mathbf{dr}' |\psi(\mathbf{k}^*, \mathbf{r}')|^2 \mathbf{S}(\mathbf{r}')$$

known from experiment

unknown

$$S(\mathbf{r}) = exp\left(-\frac{(r_{out} - \mu_{out})^2}{R_{out}^2} - \frac{r_{side}^2}{R_{side}^2} - \frac{r_{long}^2}{R_{long}^2}\right)$$

Pair wave function: (treated as weight)

$$\Psi_{pair} = \Psi_{motion} + \Psi_{FSI}$$

Interaction term

Motion of pair as whole (plane wave: exponential function, only additional phase)

Combination of the regular and singular s-wave Coulomb functions

$$\Psi(k^*, r') = \sqrt{A_C(\eta)}\left[e^{-i\vec{k}^*.\mathbf{r}^*}F(-i\eta,1,i\zeta) + f_C(\vec{k}^*)\frac{\tilde{G}(\rho,\eta)}{\mathbf{r}^*}\right]$$

Gamow factor

Confluent hypergeometric function

Strong scattering amplitude, modified by the Coulomb interaction

Adam Kisiel, *Phy.Rev.C* **81**, 064906 (2010)

Extracting the source size and emission asymmetry

$$\eta = \frac{1}{k^* a_C}$$

a_C: Bohr radius of the pair (248.52 fm for pion-kaon)

$$\zeta = k^* r^* (1 + \cos\theta^*)$$

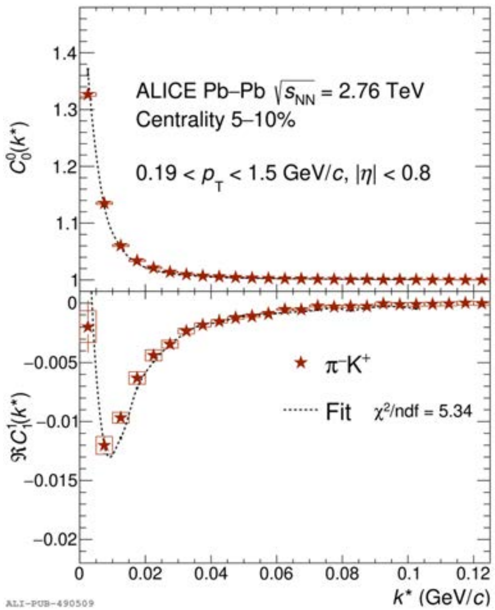
θ^*: Angle between k* and pair relative position r* in PRF

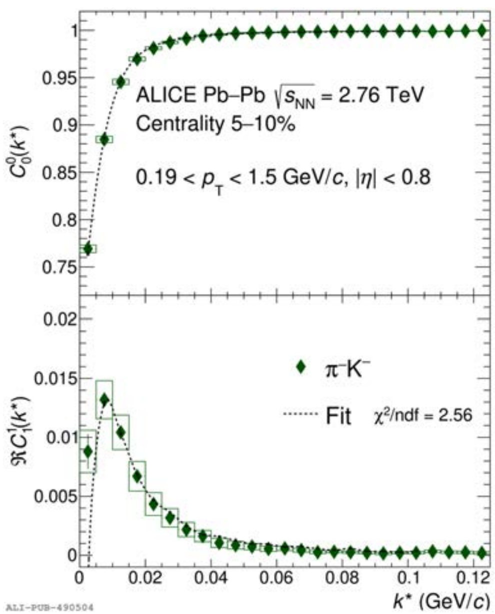
$$f_C(\vec{k}^*) = \left[\frac{1}{f_0} + \frac{1}{2} d_0 \vec{k^*}^2 - \frac{2}{a_C} h(\vec{k}^* a_C) - i \vec{k}^* A_C(\vec{k}^*)\right]^{-1}$$

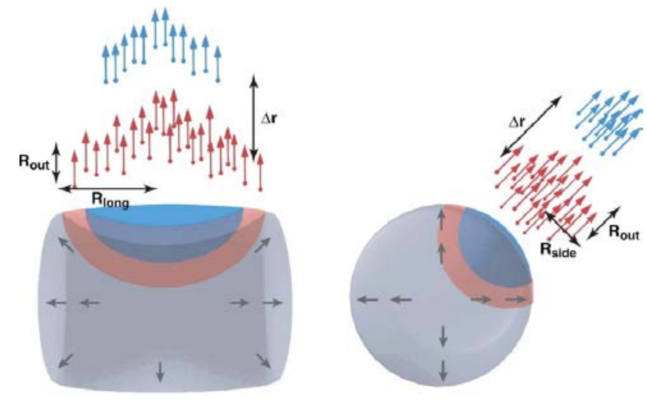
where $f_0 = 0.137$ fm for like-sign pair of pion-kaon, -0.071 fm for unlike-sign pair of pion-kaon, d_0 is the effective radius (taken to 0 for small k* where $1/f_0$ term dominates).

$$F(\alpha,1,z) = 1 + \alpha z + \alpha(\alpha+1) z^2/2!^2 + \ldots$$

Non-identical particles femtoscopy



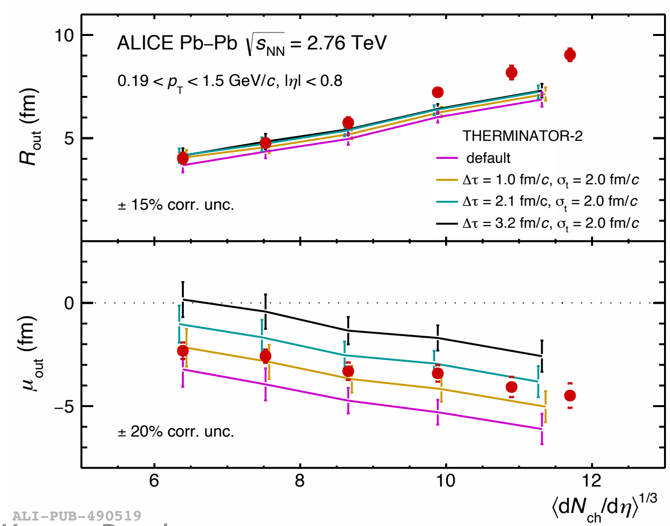




Lisa MA, et al. 2005.
Annu. Rev. Nucl. Part. Sci. 55:357–402

- 3D correlation function converted into infinite set of 1D functions in terms of spherical harmonics (Y_l^m)

$$C_l^m(\vec{k^*}) = \frac{1}{\sqrt{4\pi}} \int d\varphi d(cos\theta) C(k^*, \theta, \varphi) Y_l^m(\theta, \varphi)$$



Summary

- Average source radii decreases with increasing pair transverse momentum
- Average source radii increase with increasing system size/multiplicity
- Lifetime of the fireball increases from periferal to central collisions
- Finite emission asymmetry observed between pions and kaons which shows pions are emitted later than kaons
- It is expected in a system with strong collectivity which includes flow of resonances (consistent with model predictions, e.g. Therminator2 coupled with viscous hydrodynamics)
- Source size and emission asymmetry increase from peripheral to central collisions
- Results may suggest a 2.1 fm/*c* delay in emission time which means different particle species freeze-out at different times

THANK YOU

Correlation from strong interaction

$$C(q) = \int S(r)|\psi(q,r)|^2 d^4 r \qquad q = 2k^*$$

measured correlation

emission function (source size/shape)

pair wave function (includes cross section)

$$\psi = exp(-ik * r) + f\frac{exp(ik^* r)}{r}$$ s-wave scattering approximation

$$f^{-1}(k^*) = \frac{1}{f_0} + \frac{1}{2}d_0 k^{*2} - ik^*$$ effective range approximation

For only Strong Final State Interaction:

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f^S(k^*)}{R}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}R}\right) + \frac{2\Re f^S(k^*)}{\sqrt{\pi}R}F_1(2k^*R) - \frac{\Im f^S(k^*)}{R}F_2(2k^*R)\right]$$

spin fractions

Lednicky, Lyuboshitz, Sov. J. Nucl. Phys., 35, 770 (1982)

- The correlation function is finally characterised by **three parameters**:
  - **radius** R, **scattering length** f_0 and **effective radius** d_0
  - Cross-section **σ** (at low k*) is simply: **σ** = $4\pi|f|^2$

$$F_1(z) = \int_0^z \chi e^{\chi^2 - z^2}/z dz$$
$$F_2(z) = (1 - e^{-z})/z$$