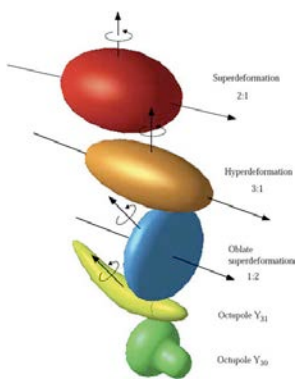
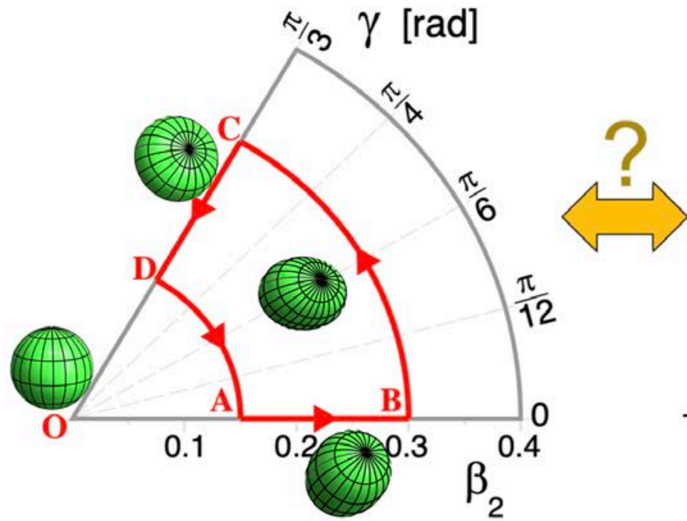
Imaging the nuclear structure and initial condition of heavy-ion collisions across the nuclei chart

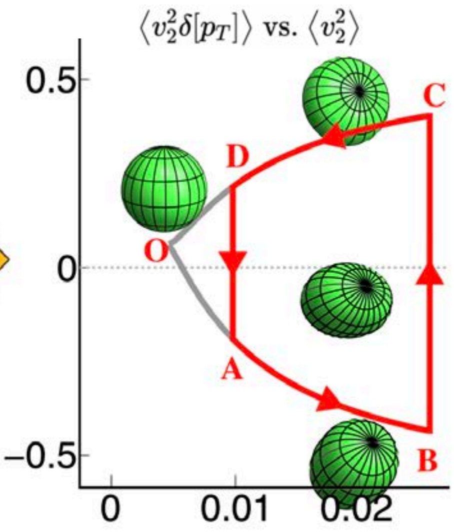
Jiangyong Jia

Nuclear structure

High-energy collisions







based partially on arXiv:2209.11042

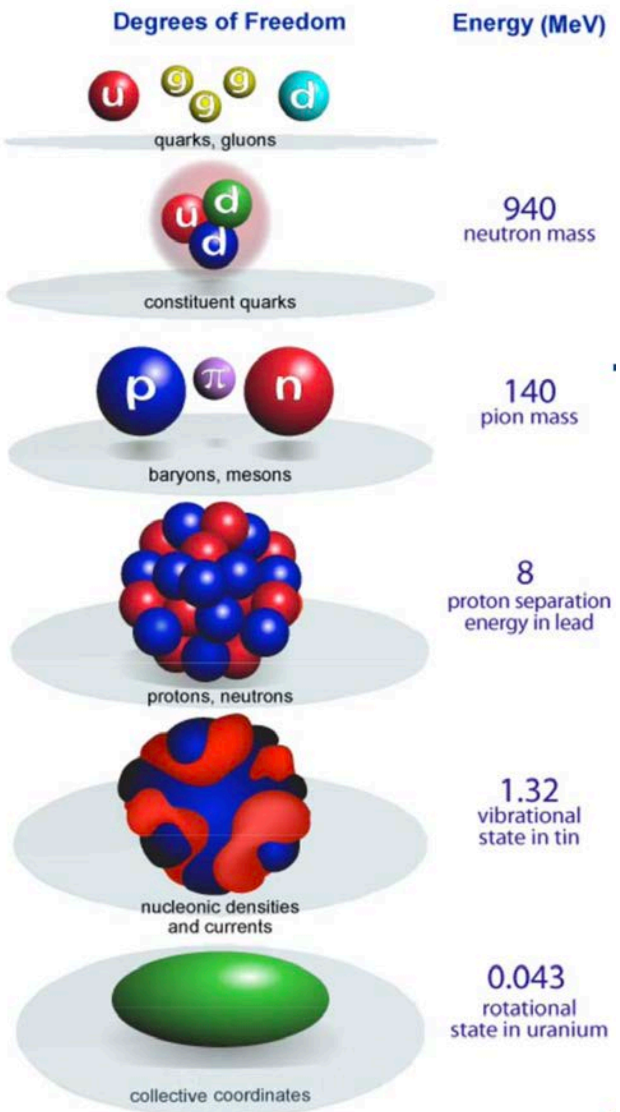
Imaging the initial condition of heavy-ion collisions and nuclear structure across the nuclide chart

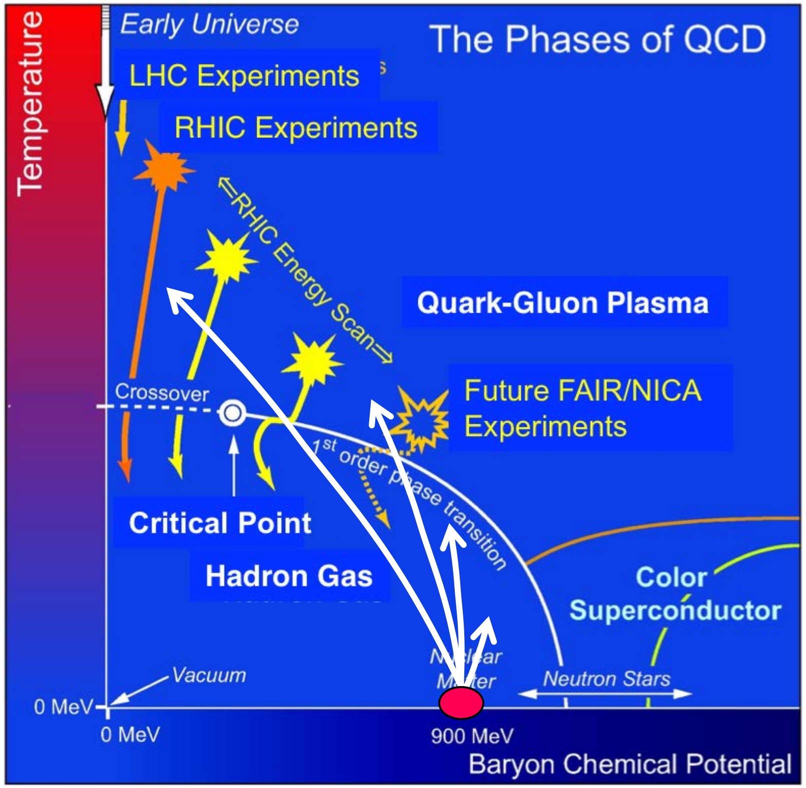
Landscape of nuclear physics

Quark-gluon plasma

hadrons

nuclei

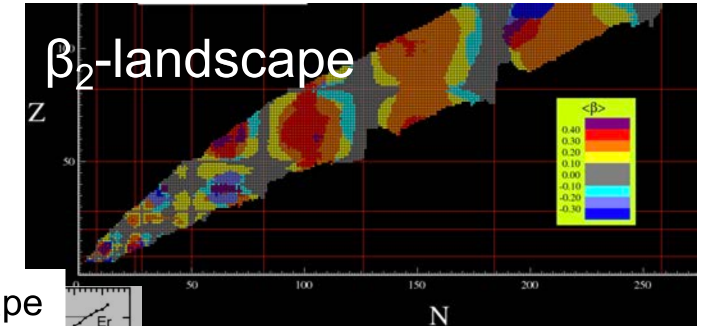


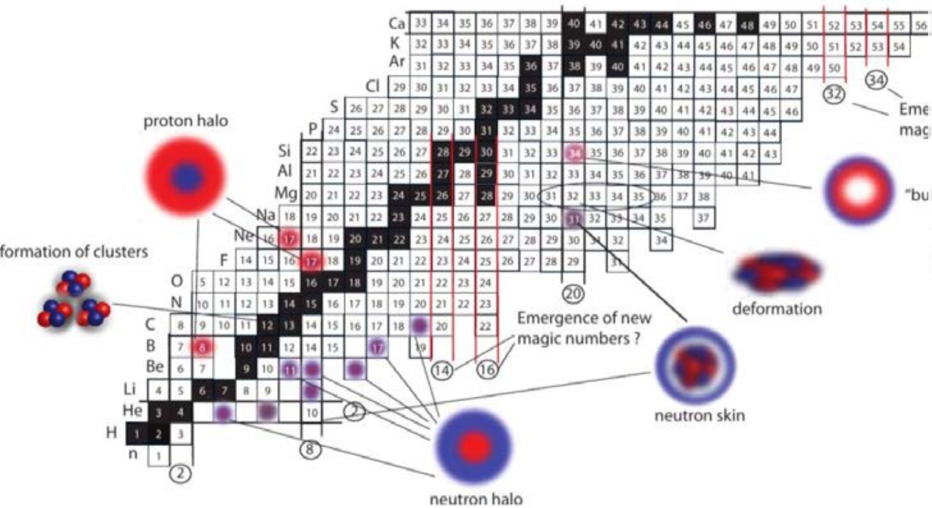


Most nuclear experiments starts with nuclei

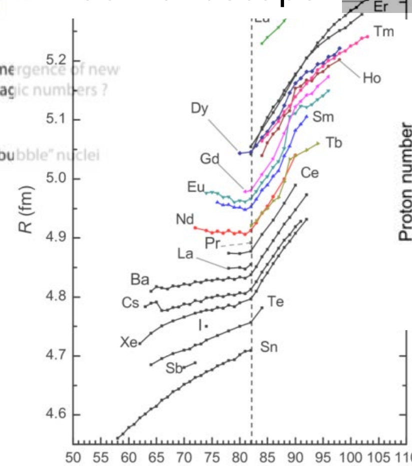
Rich structure of atomic nuclei

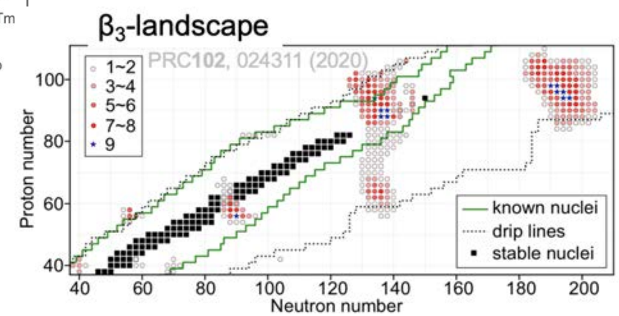
- Collective phenomena of many-body quantum system
  - clustering, halo, skin, bubble…
  - quadrupole/octupole/hexdecopole deformations
  - Nontrivial evaluation with N and Z.

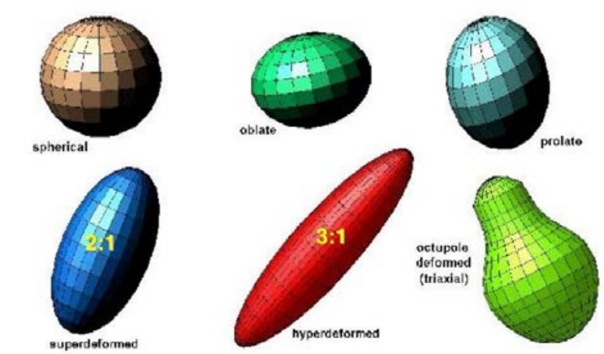




Radii-landscape

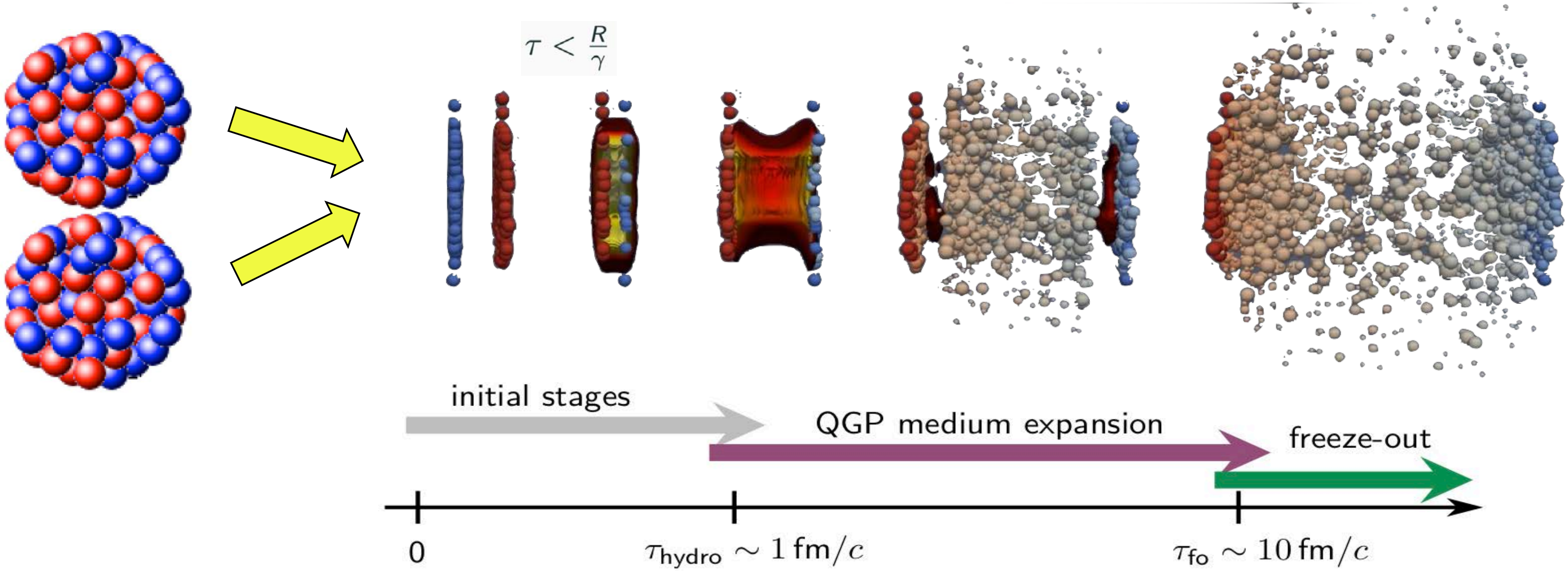






- Understanding via effective nuclear theories
  - Lattice, Ab.initio (starting from NN interaction)
  - Shell models (configuration interaction)
  - DFT models (non-relativistic and covariant)

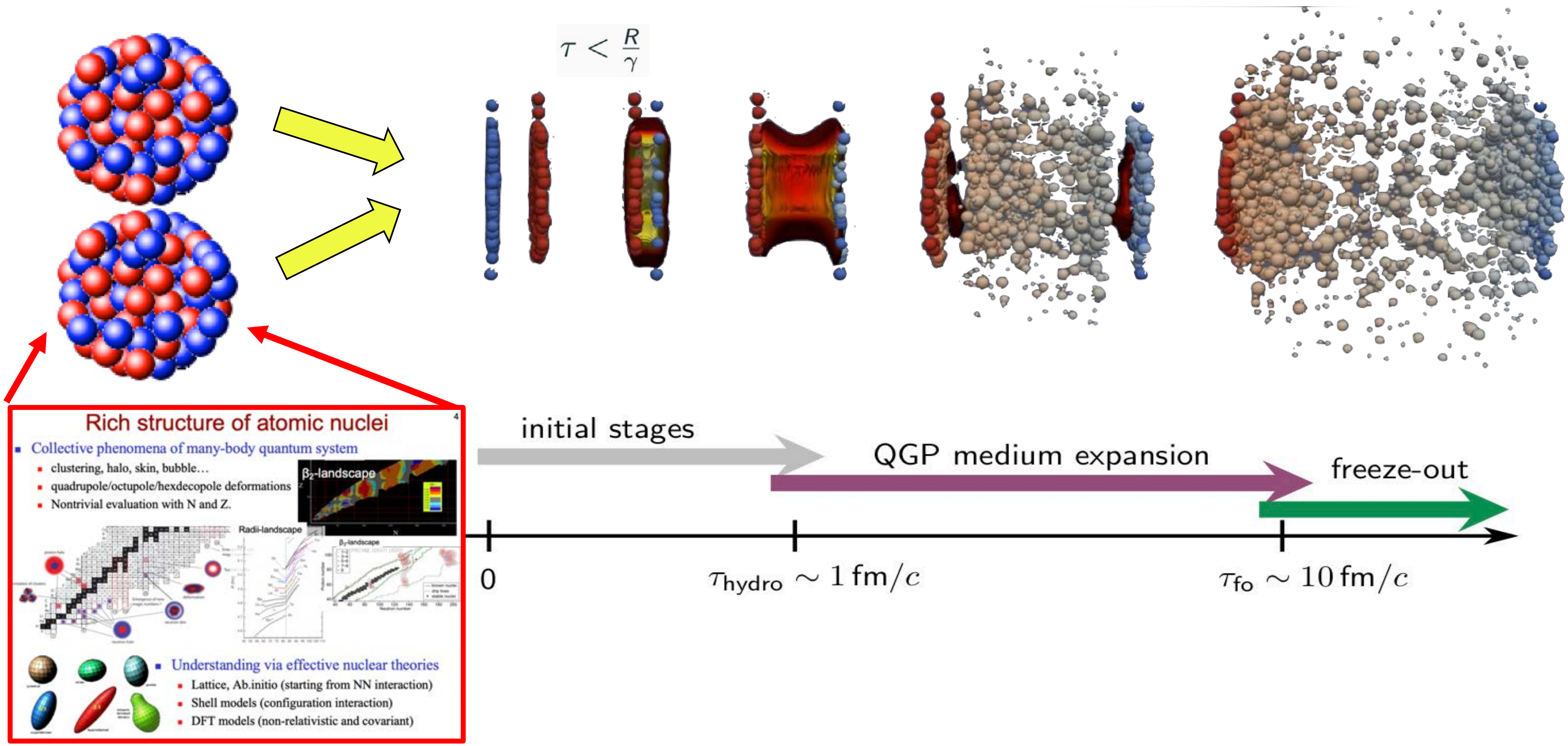
High-energy heavy ion collision



400 nucleons → 30000 hadrons

in 10^{-23} seconds

High-energy heavy ion collision



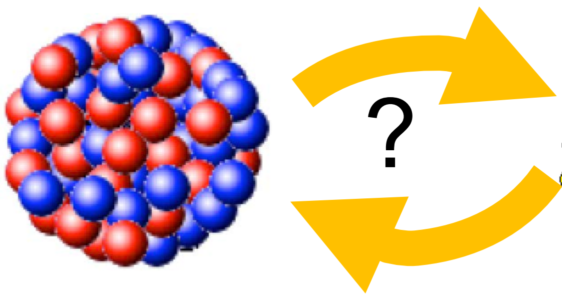
1) Are nuclear structures important for HI initial condition and final state evolution?

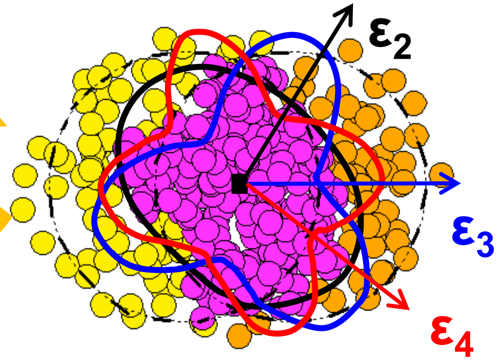
2) What HI experimental observables can be used to infer structure information?

3) Can HI provides competitive constraints on nuclear shape and radial profile? can consideration of nuclear structure improves understanding of HI initial condition?

From nuclear structure to Quark Gluon Plasma

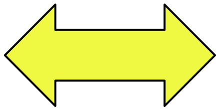
Nucleus | Initial condition | Final state

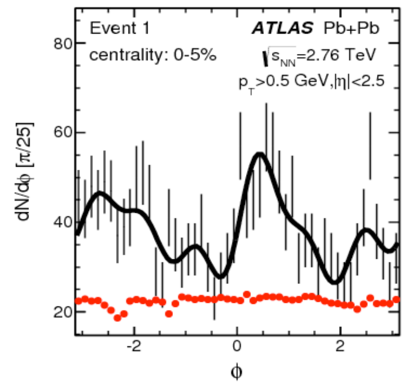




Fixed impact parameter

hydro

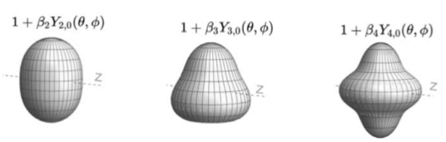




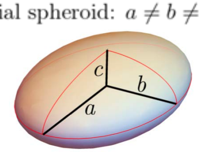
Nuclear structure

$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\Big(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}] + \beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m} + \beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m}\Big)$$



Triaxial spheroid: $a \neq b \neq c$.



$0 \leq \gamma \leq \pi/3$

Initial volume	Initial Size	Initial Shape
N_{part}	$R_\perp^2 \propto \langle r_\perp^2 \rangle$,	$\mathcal{E}_2 \propto \left\langle r_\perp^2 e^{i2\phi}\right\rangle$
		$\mathcal{E}_3 \propto \left\langle r_\perp^3 e^{i3\phi}\right\rangle$
		$\mathcal{E}_4 \propto \left\langle r_\perp^4 e^{i4\phi}\right\rangle$
		...

Multiplicity	Radial Flow	Harmonic Flow
N_{ch}	$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$	

High energy: approx. linear response in each event: $\tau < \frac{R}{\gamma}$

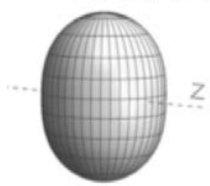
$$N_{ch} \propto N_{part} \qquad \frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n$$

Collective structure of nuclei

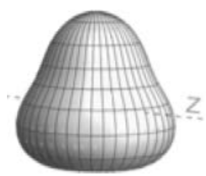
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}]+\beta_3\sum_{m=-3}^{3}\alpha_{3,m}Y_{3,m}+\beta_4\sum_{m=-4}^{4}\alpha_{4,m}Y_{4,m}\right)$$

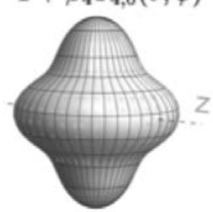
Quadrupole: $1+\beta_2 Y_{2,0}(\theta,\phi)$

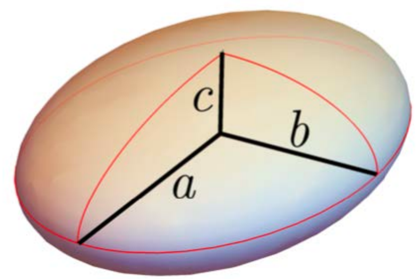


Octupole: $1+\beta_3 Y_{3,0}(\theta,\phi)$



Hexadecapole: $1+\beta_4 Y_{4,0}(\theta,\phi)$





$0 \le \gamma \le \pi/3$

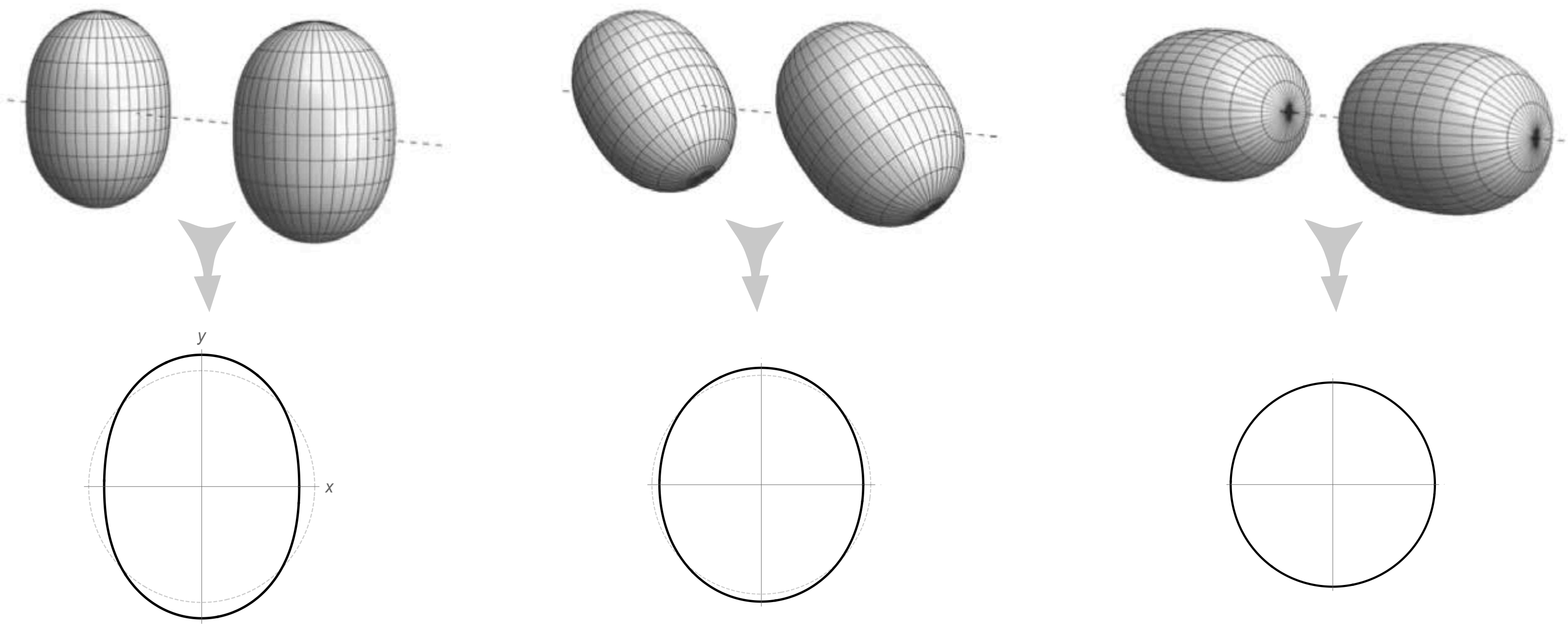
Prolate: a=b<c → β_2, γ=0

Oblate: a<b=c → β_2, γ=π/3

Triaxial: a<b<c → β_2, γ=π/6

How deformation influence HI initial state

Body-Body $\qquad \mathcal{E}_2 = \epsilon_2 e^{i2\Phi} \propto \left\langle \boldsymbol{r}_\perp^2 e^{i2\phi} \right\rangle \qquad$ Tip-Tip



$\varepsilon_2 \sim 0.95\beta_2 \qquad \varepsilon_2 \sim 0.48\beta_2 \qquad \varepsilon_2 \sim 0$

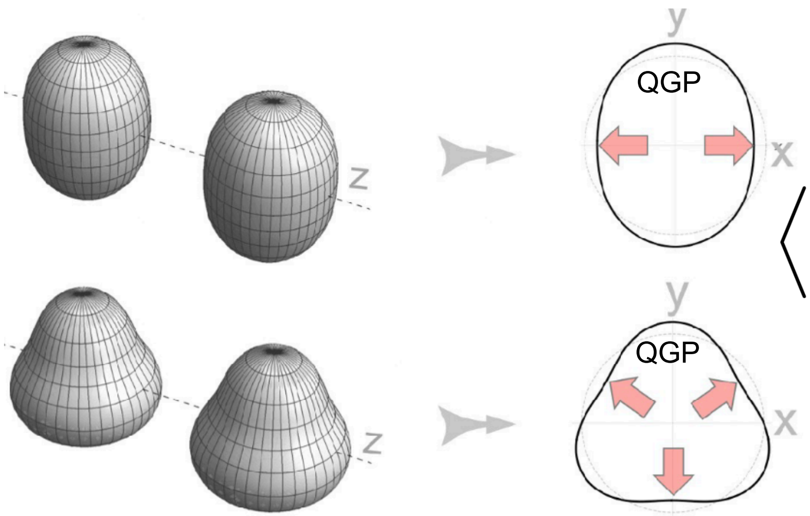
$$\boldsymbol{\epsilon}_2 = \underbrace{\boldsymbol{\epsilon}_0}_{\text{undeformed}} + \underbrace{\boldsymbol{p}(\Omega_1, \Omega_2)}_{\text{phase factor}} \beta_2 + \mathcal{O}\left(\beta_2^2\right) \Longrightarrow \left\langle \epsilon_2^2 \right\rangle \approx \left\langle \epsilon_0^2 \right\rangle + 0.2\beta_2^2$$

Shape depends on Euler angle Ω=φθψ

Expected structure dependencies

arXiv:2106.08768

Central collisions

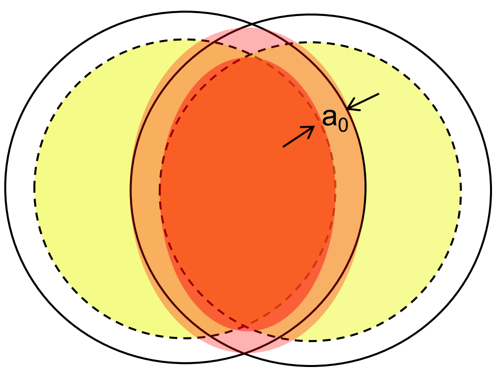


$$\langle v_2^2 \rangle \approx a_2 + b_2\beta_2^2 + c_3\beta_3^2$$

$$\left\langle (\delta p_T/p_T)^2 \right\rangle \approx a_0 + b_0\beta_2^2 + c_0\beta_3^2$$

$$\langle v_3^2 \rangle \approx a_3 + b_3\beta_3^2$$

Non-Central collisions



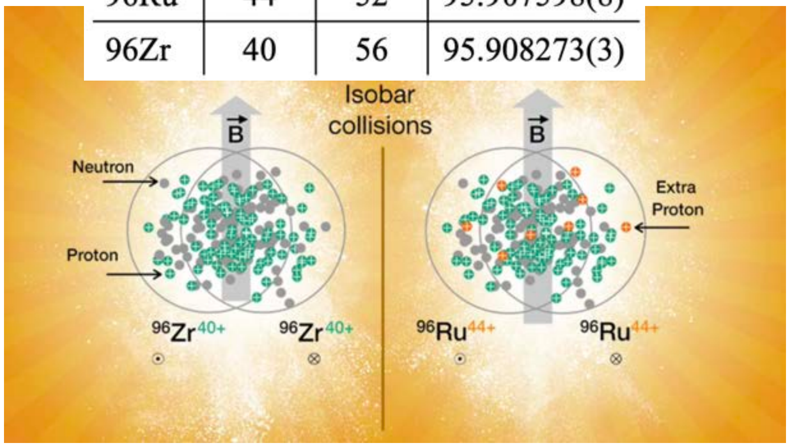
The shape and size the overlap, therefore v_2 and p_T, also depend on diffuseness a_0 and radius R_0

At fixed N_{part} $a_0\searrow$ ⇨ $v_2\nearrow$ $p_T\nearrow$

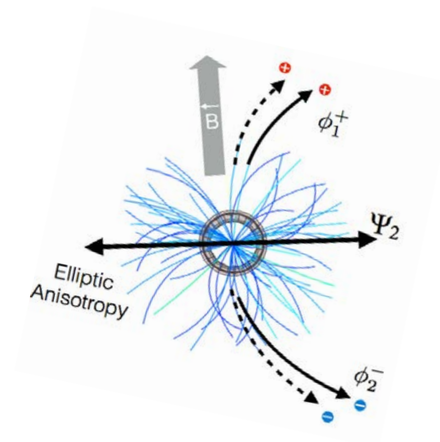
$R_0\searrow$ ⇨ $p_T\nearrow$

Isobar collisions at RHIC

	Z	N	mass
96Ru	44	52	95.907598(8)
96Zr	40	56	95.908273(3)



arXiv:2109.00131



Voloshin, hep-ph/0406311

- Designed to search for the chiral magnetic effect: strong P & CP violation of QCD in the presence of EM field. Turns out the CME signal is small, and isobar-differences are dominated by the nuclear structure differences.

- <0.4% precision is achieved in ratio of many observables between ^{96}Ru+ ^{96}Ru and ^{96}Zr+^{96}Zr systems→ precision imaging tool

Isobar collisions as a precision tool

- A key question for any HI observable O:

$$\frac{O_{^{96}\mathrm{Ru}+^{96}\mathrm{Ru}}}{O_{^{96}\mathrm{Zr}+^{96}\mathrm{Zr}}} \stackrel{?}{=} 1$$

Deviation from 1 must has origin in the nuclear structure, which impacts the initial state and then survives to the final state.

- Expectation

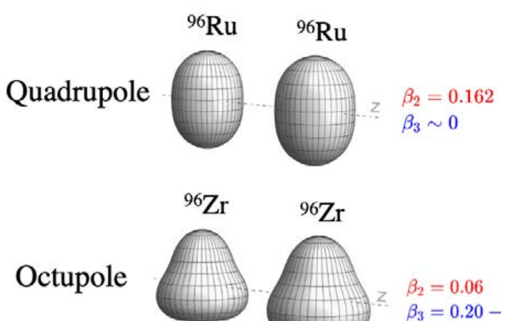
$$\rho(r,\theta,\phi) \propto \frac{1}{1+e^{[r-R_0(1+\beta_2 Y_2^0(\theta,\phi)+\beta_3 Y_3^0(\theta,\phi))]/a_0}}$$

2109.00131

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Only probes isobar differences



J. Jia, aXiv:2106.08768

Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

Relate to neutron skin: $\Delta r_{\mathrm{np}} = \langle r_n\rangle^{1/2} - \langle r_p\rangle^{1/2}$

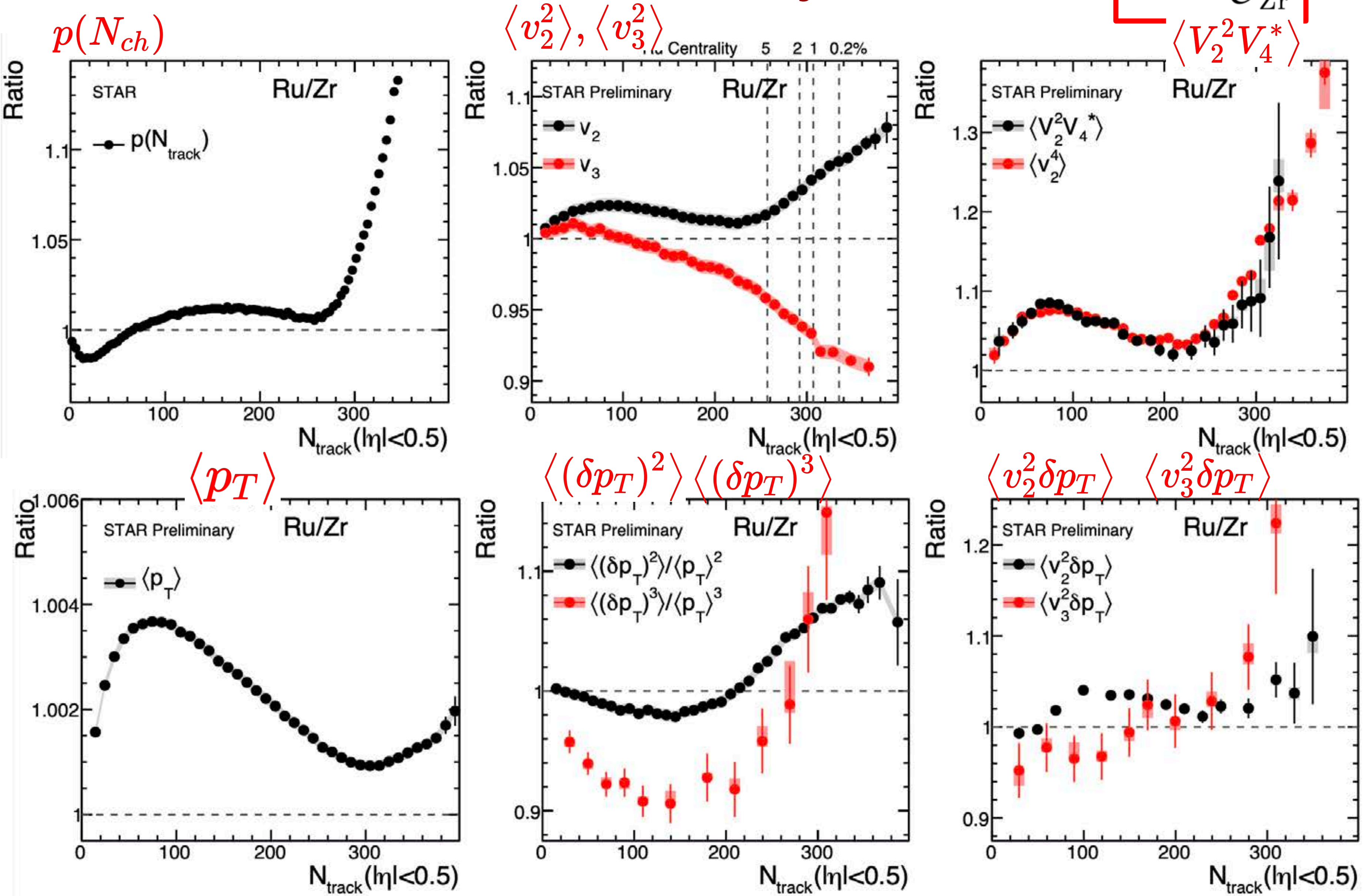
charge

$$\Delta r_{np,\mathrm{Ru}} - \Delta r_{np,\mathrm{Zr}} \propto (R_0\Delta R_0 - R_{0p}\Delta R_{0p}) + 7/3\pi^2(a\Delta a - a_p\Delta a_p)$$

mass

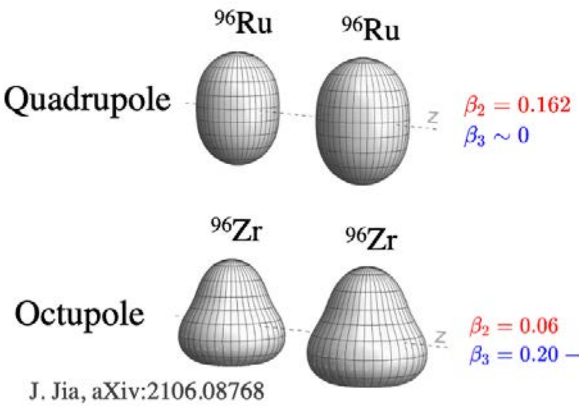
Structure influences everywhere

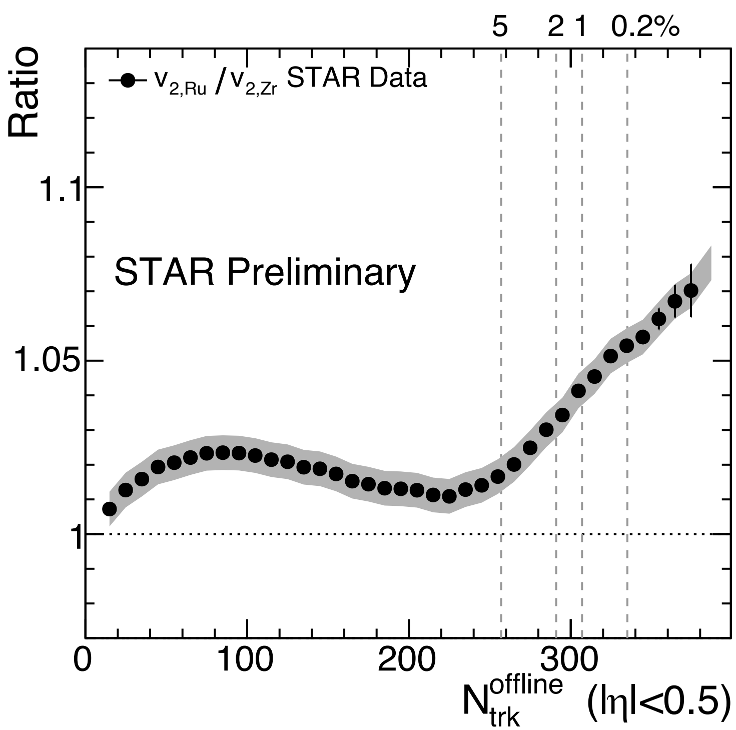
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}}$$

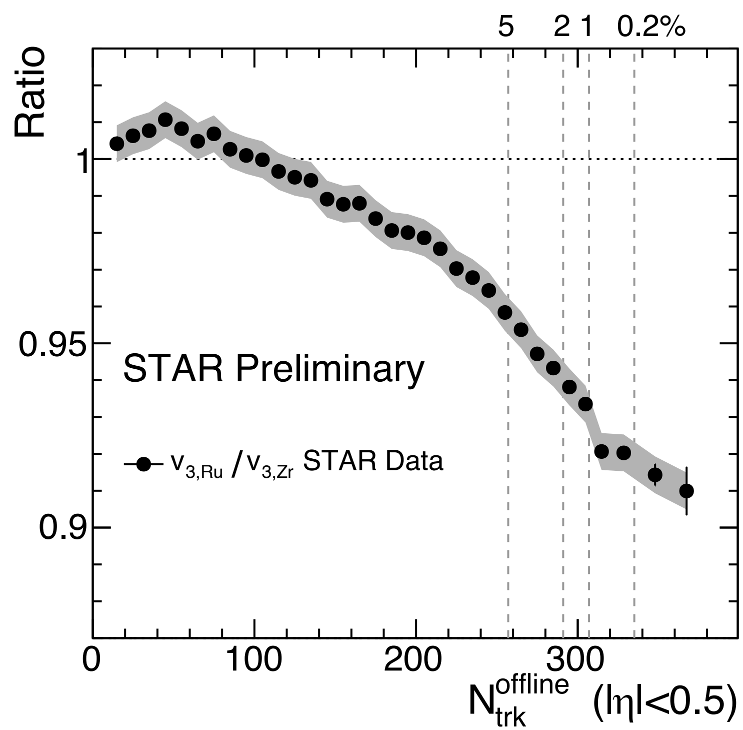


We can ask the same question for isobar ratios for hard probes

Nuclear structure via v_n-ratio

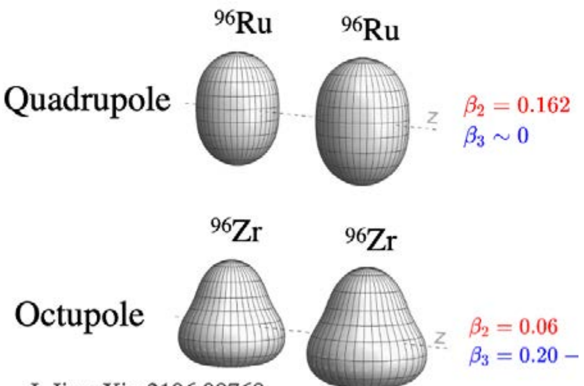






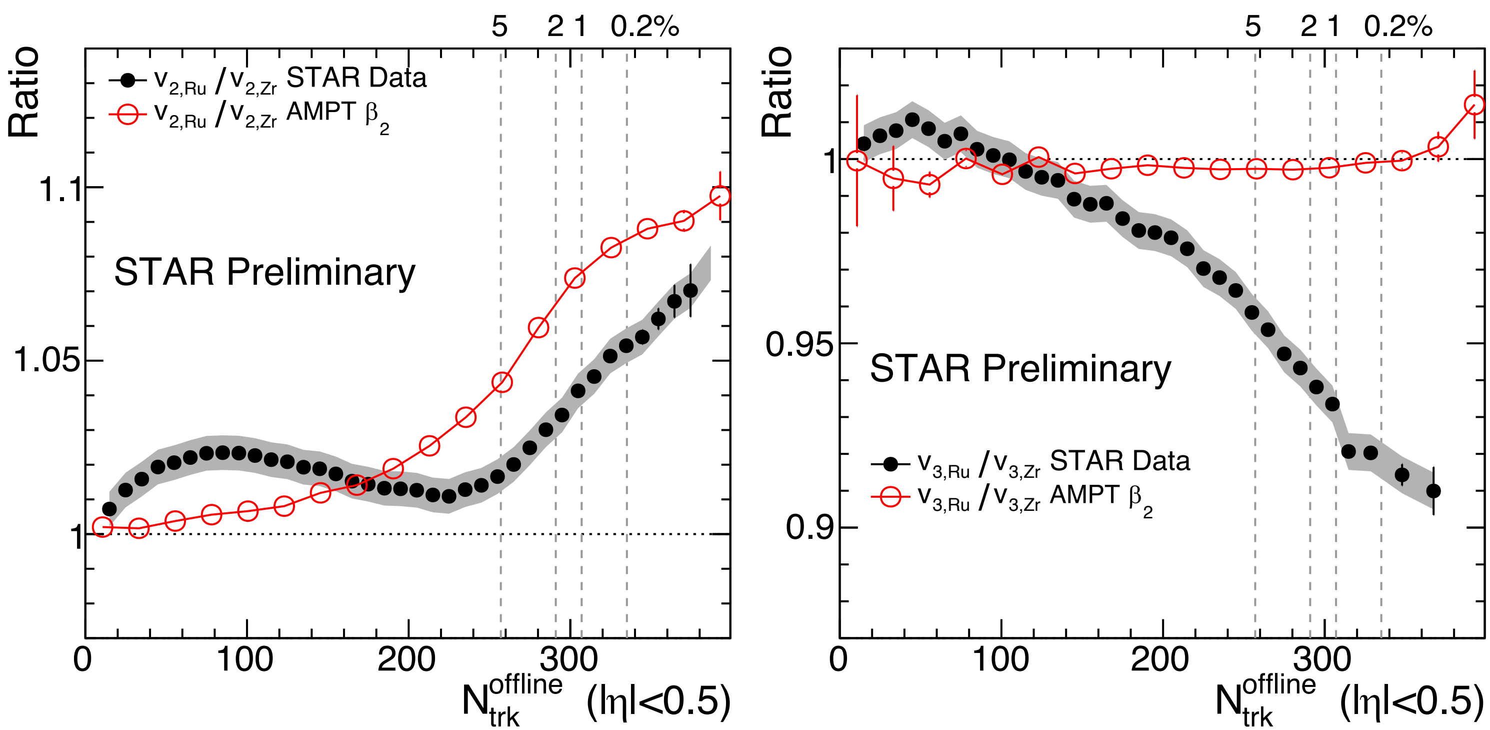
Simultaneously constrain these parameters using different N_{ch} regions

Nuclear structure via v_n-ratio



J. Jia, aXiv:2106.08768

- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio



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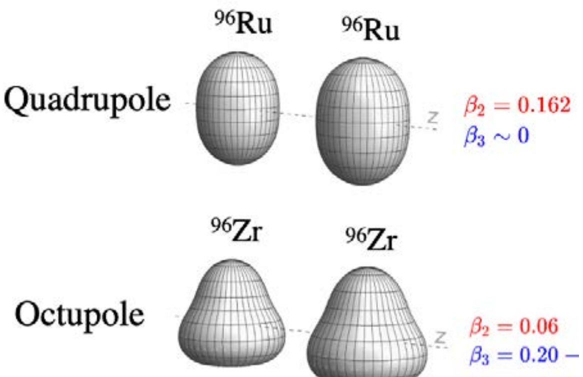
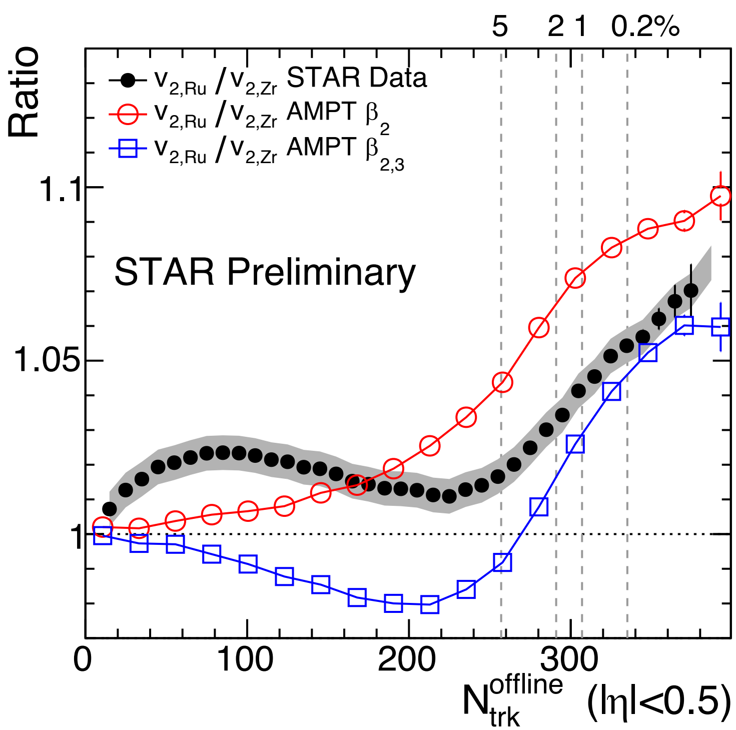
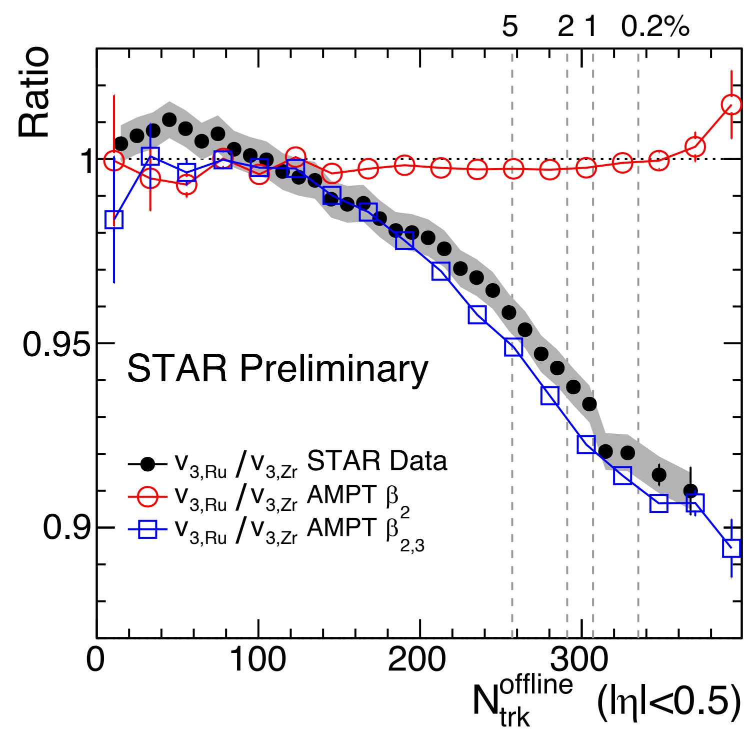
Nuclear structure via v_n-ratio



J. Jia, aXiv:2106.08768

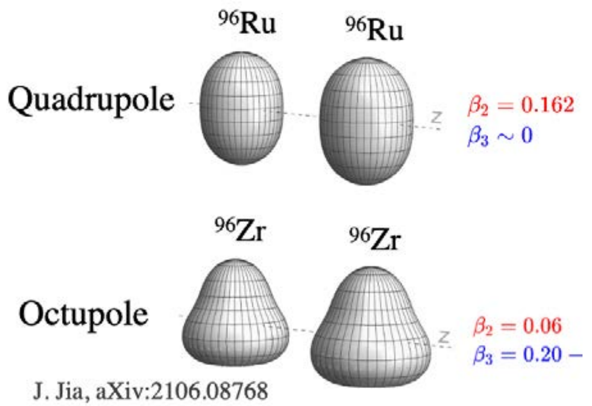
- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio



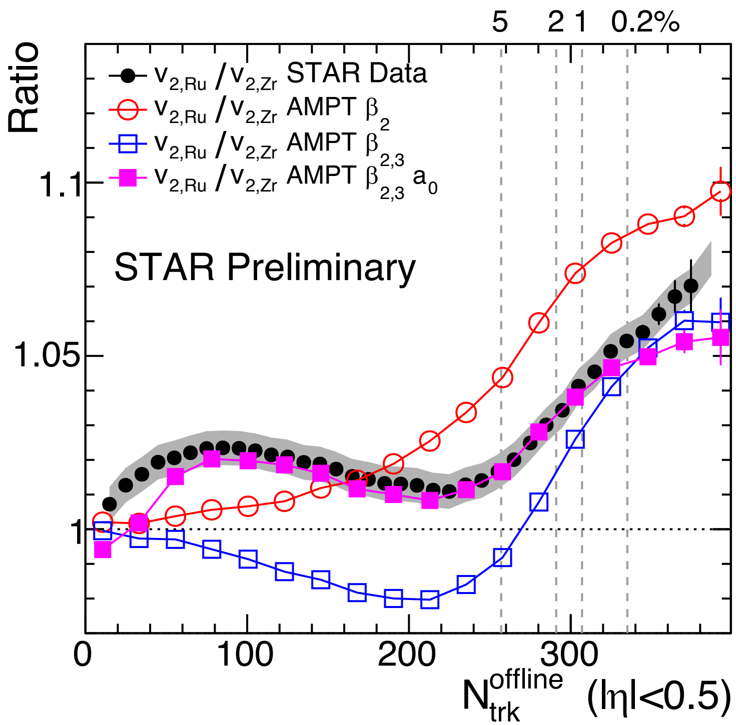


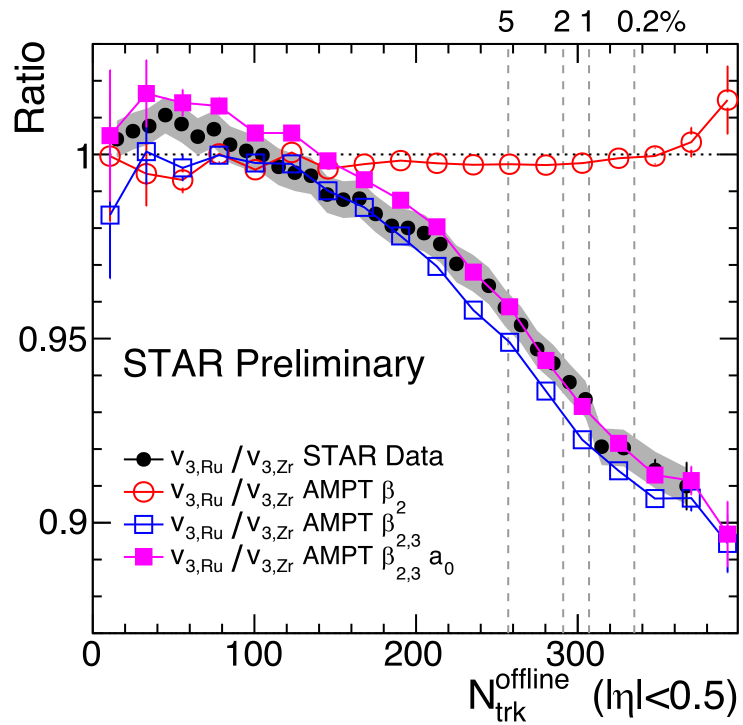
Simultaneously constrain these parameters using different N_{ch} regions

Nuclear structure via v_n-ratio



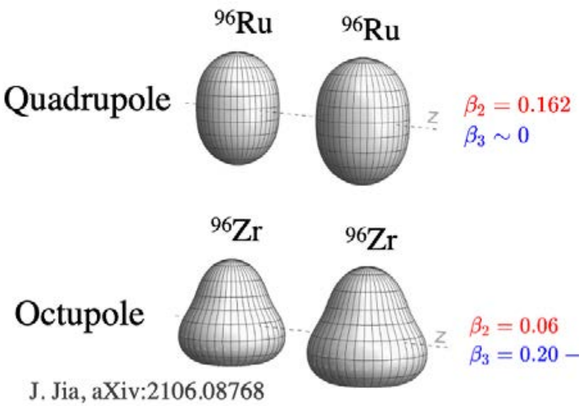
- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio
- Δa_0= -0.06fm increase v_2 mid-central, small influ. on v_3.





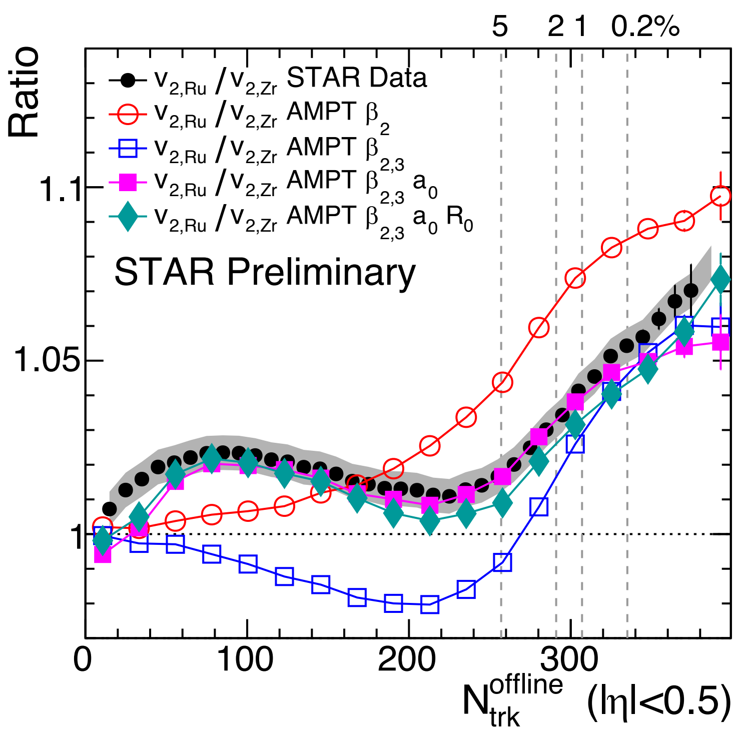
Simultaneously constrain these parameters using different N_{ch} regions

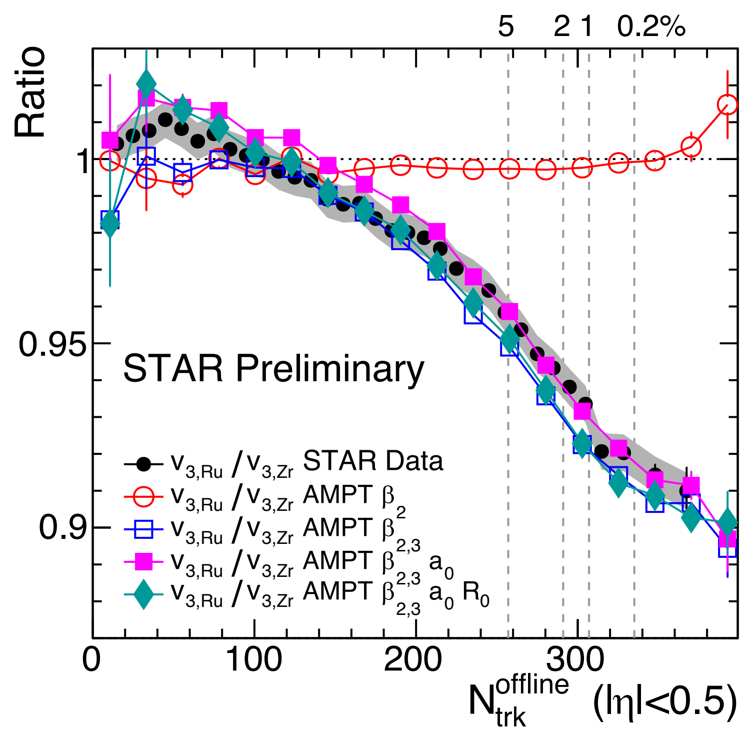
Nuclear structure via v_n-ratio



- β_{2Ru}~0.16 increase v_2, no influence on v_3 ratio
- β_{3Zr}~0.2 decrease v_2 in mid-central, decrease v_3 ratio
- Δa_0= -0.06fm increase v_2 mid-central, small influ. on v_3.
- Radius ΔR_0=0.07fm only slightly affects v_2 and v_3 ratio.

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1 \Delta\beta_2^2 + c_2 \Delta\beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

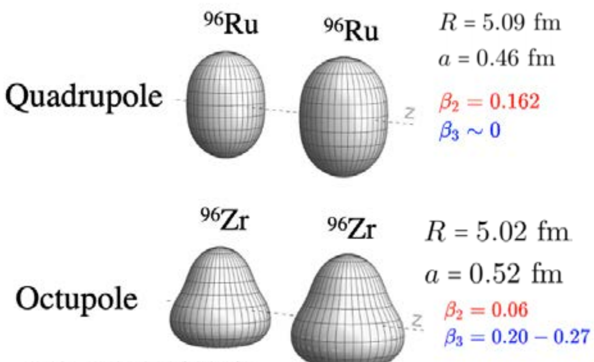




Simultaneously constrain these parameters using different N_{ch} regions

Nuclear structure via $p(N_{ch})$, <pT>-ratio

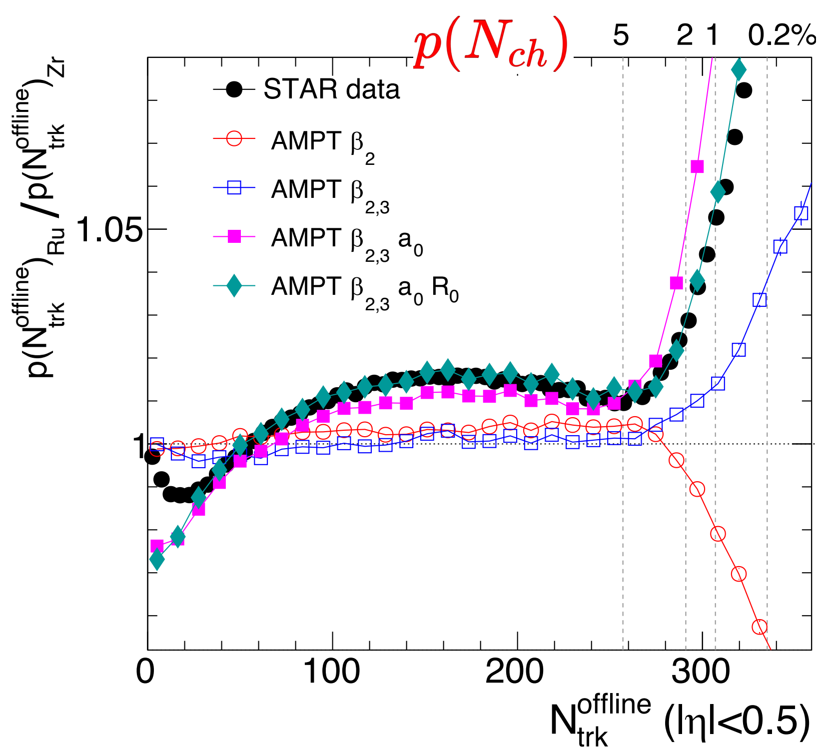
Earlier studies on this from H.Li, H.J Xu, PRL125, 222301 (2020) arXiv:2111.14812



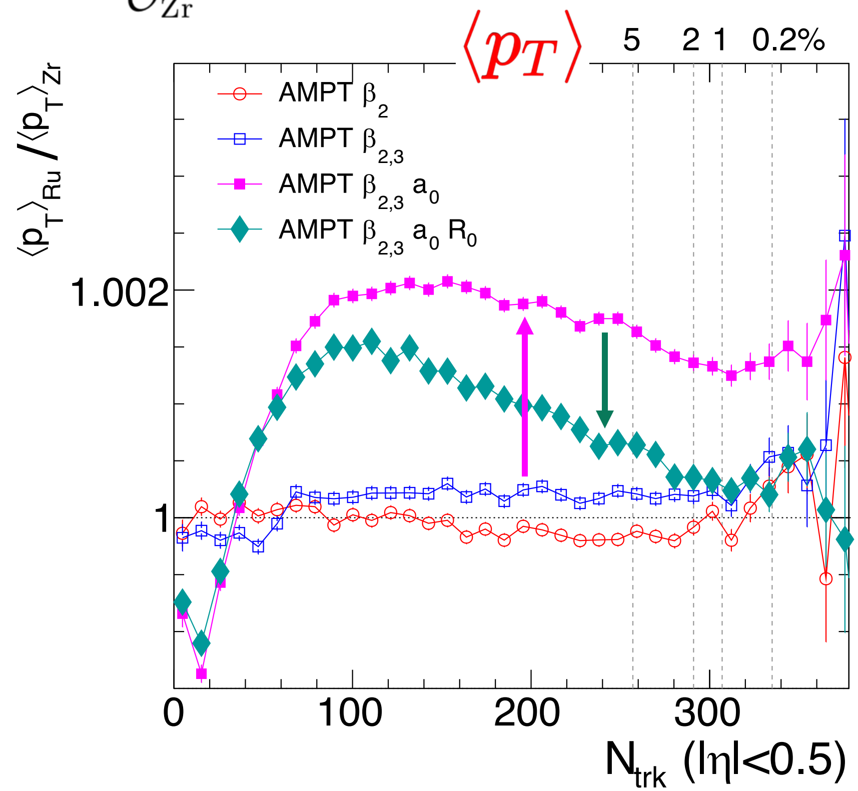
J. Jia, aXiv:2106.08768

- For Nch ratio:
  - β_{2Ru}~0.16 decrease ratio, increase after considering β_{3Zr}~0.2
  - The bump structure in non-central region from Δa_0 and ΔR_0
- For <pT> ratio:
  - Strong influence from Δa_0 and ΔR_0

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\rm Ru}}{\mathcal{O}_{\rm Zr}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$



Δa_0 and ΔR_0 influences add up



Δa_0 and ΔR_0 influences cancel

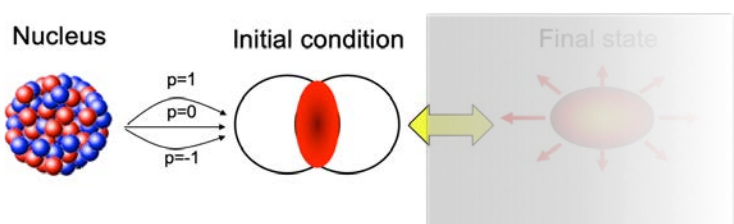
Isobar ratios not affected by final state

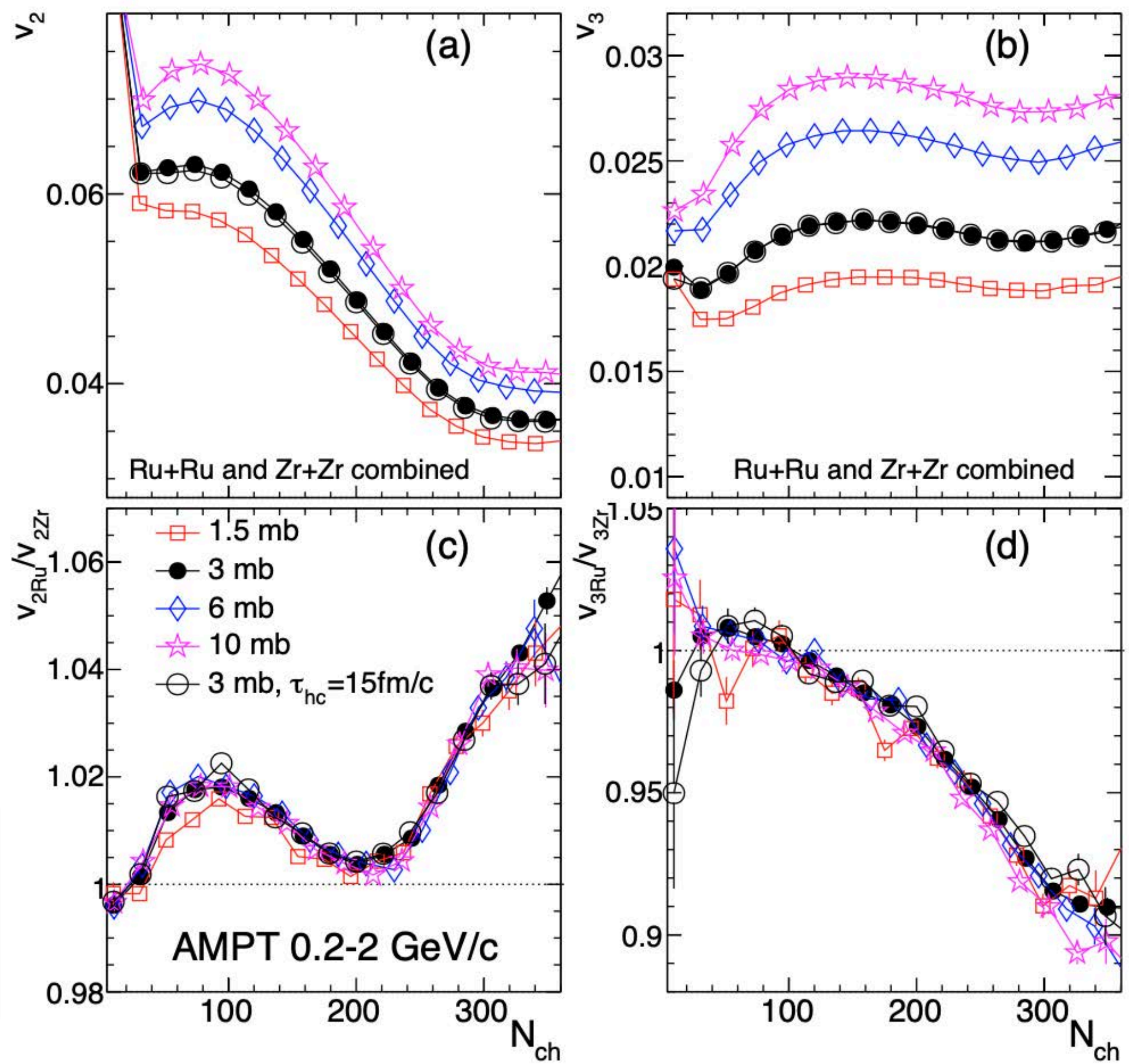
- Vary the shear viscosity via partonic cross-section
  - Flow signal change by 30-50%, the v_n ratio unchanged.

$$v_n = k_n \varepsilon_n$$

$$\frac{v_{n,\mathrm{Ru}}}{v_{n,\mathrm{Zr}}} \approx \frac{\varepsilon_{n,\mathrm{Ru}}}{\varepsilon_{n,\mathrm{Zr}}}$$

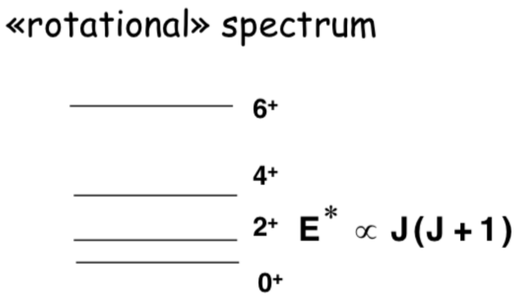
Robust probe of initial state!

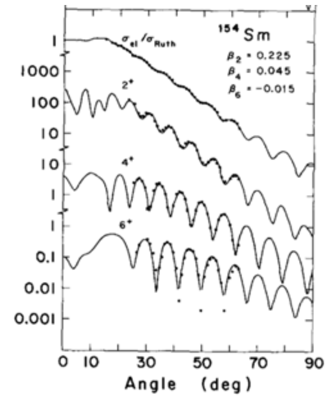


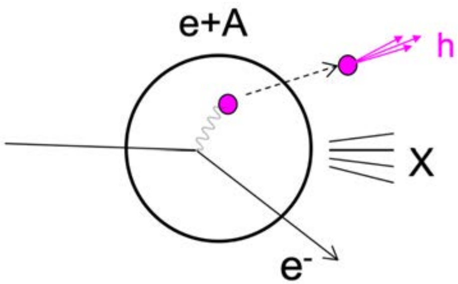


Low-energy vs high-energy HI method

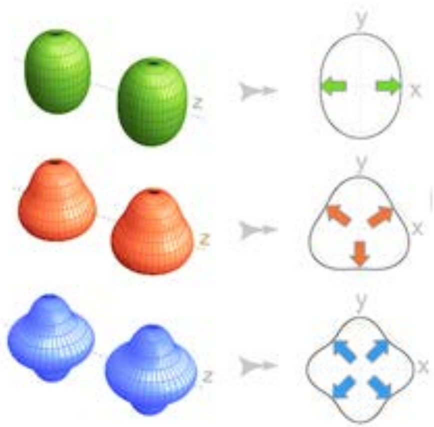
- Shape from B(En), radial profile from e+A or ion-A scattering



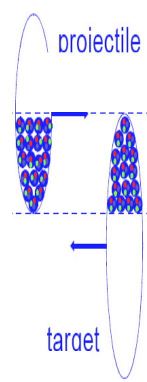


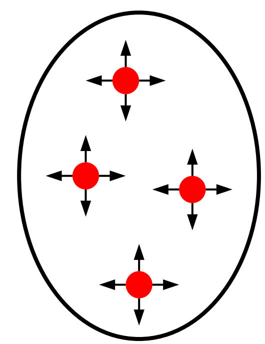


- Shape frozen in crossing time ($<10^{-24}$s), probe entire mass distribution via multi-point correlations.



Collective flow response to nuclear structure



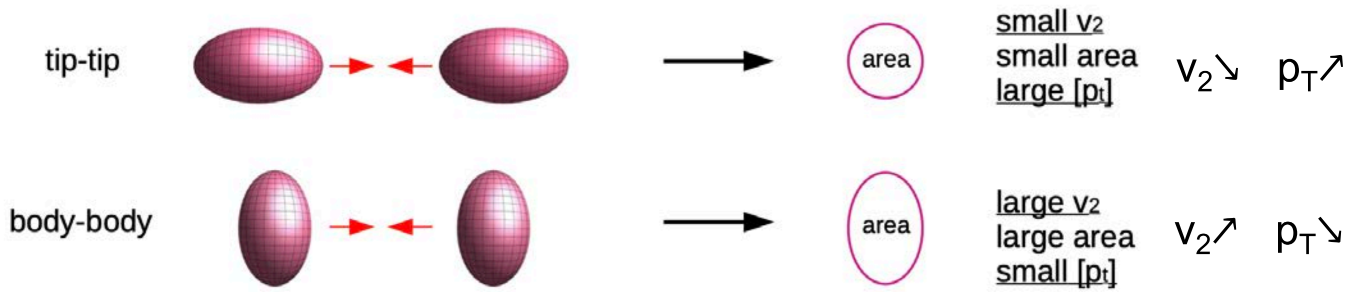


$$S(\mathbf{s}_1, \mathbf{s}_2) \equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle$$
$$= \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle \rho(\mathbf{s}_1)\rangle\langle \rho(\mathbf{s}_2)\rangle.$$

Triaxility

$$R(\theta, \phi) = R_0 \Big(1 + \beta_2 [\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]$$

1910.04673, 2004.14463



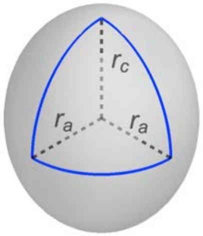
Need 3-point correlators to probe the 3 axes

$$\left\langle v_2^2 \delta p_{\mathrm{T}} \right\rangle \sim -\beta_2^3 \cos(3\gamma) \qquad \left\langle (\delta p_{\mathrm{T}})^3 \right\rangle \sim \beta_2^3 \cos(3\gamma)$$

2109.00604

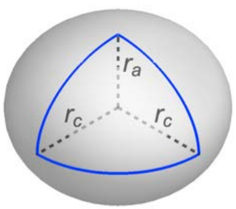
Prolate

$\beta_2 = 0.25, \cos(3\gamma) = 1$



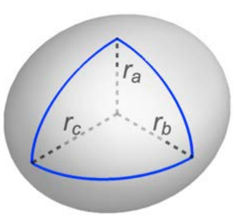
Triaxial

$\beta_2 = 0.25, \cos(3\gamma) = 0$



Oblate

$\beta_2 = 0.25, \cos(3\gamma) = -1$



Triaxility $R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$

1910.04673, 2004.14463

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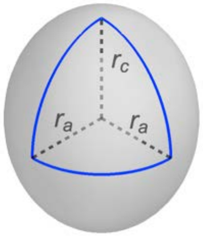

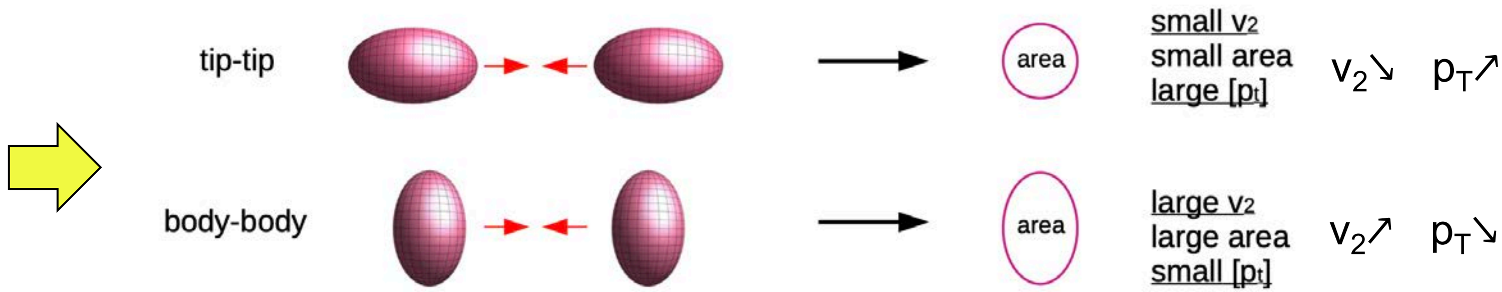

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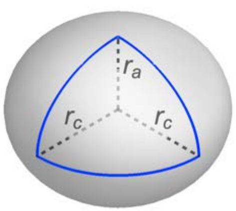

Oblate

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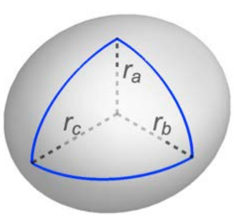
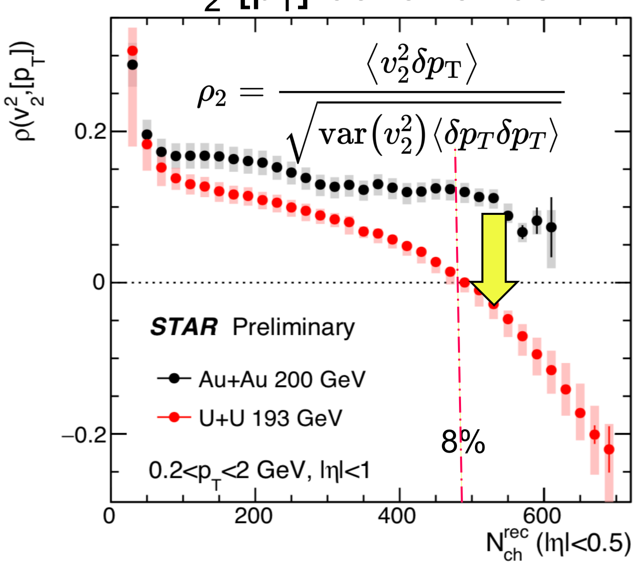
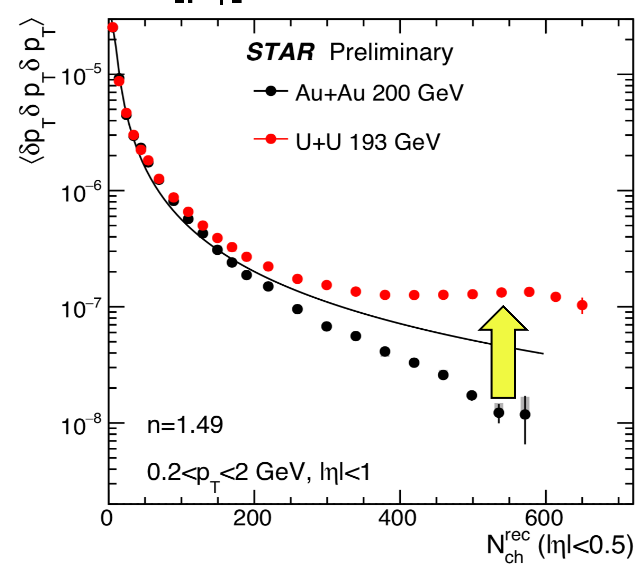


Compare U+U vs Au+Au: $\beta_{2U} \sim 0.28$, $\beta_{2Au} \sim 0.13$:

v_2-$[p_T]$ covariance



$[p_T]$ skewness

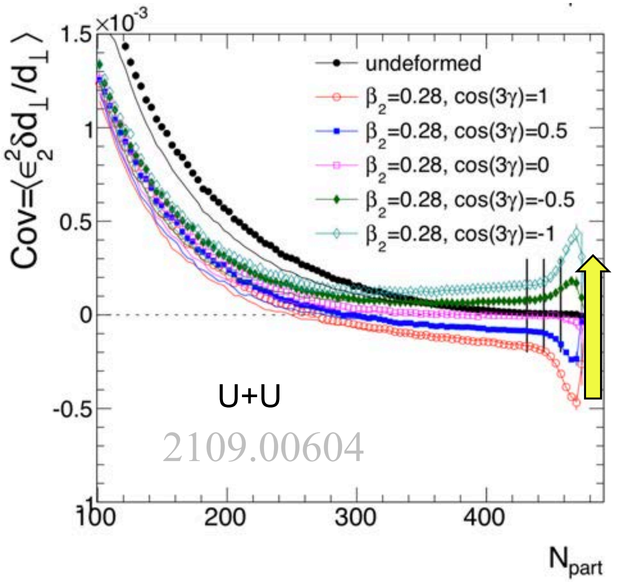


Influence of triaxiality: Glauber model

Skewness super sensitive

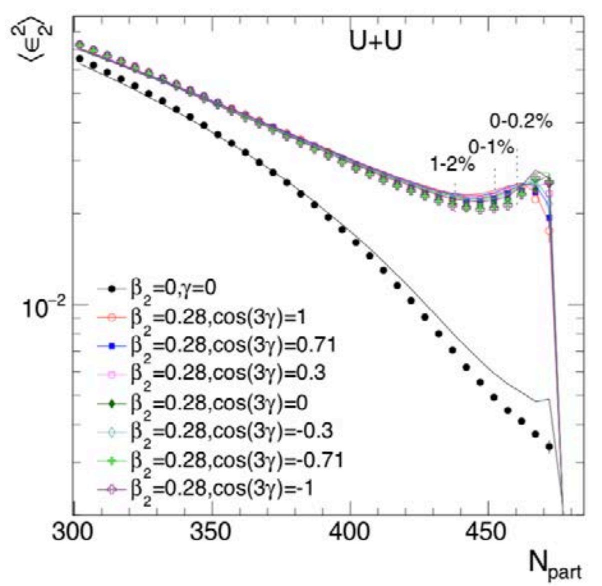
Described by

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle \propto \langle v_2^2 \delta p_{\mathrm{T}} \rangle \propto a + b\cos(3\gamma)\beta_2^3$$



variances insensitive to γ

$$\langle \varepsilon_2^2 \rangle \propto \langle v_2^2 \rangle \propto a + b\beta_2^2$$



Use variance to constrain β_2, use skewness to constrain γ

(β_2,γ) diagram in heavy-ion collisions

The (β_2,γ) dependence in 0-1% U+U Glauber model can be approximated by:

$$d_\perp \propto 1/R_\perp$$

$$\langle\varepsilon_2^2\rangle \approx [0.02+\beta_2^2]\times 0.235$$

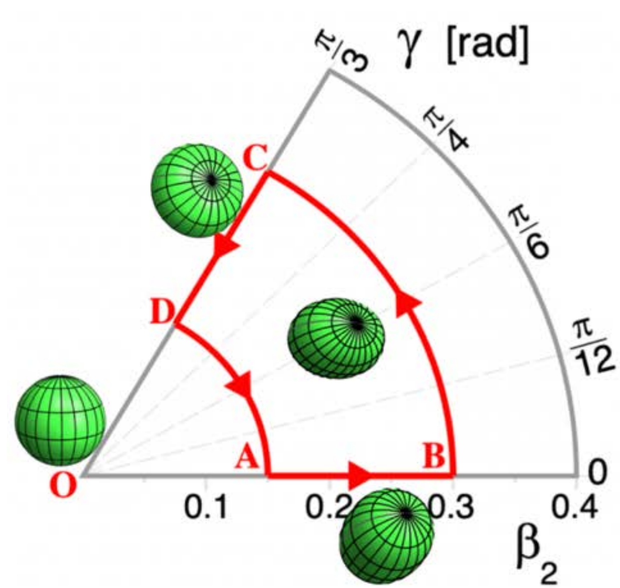
$$\langle(\delta d_\perp/d_\perp)^2\rangle \approx [0.035+\beta_2^2]\times 0.0093$$

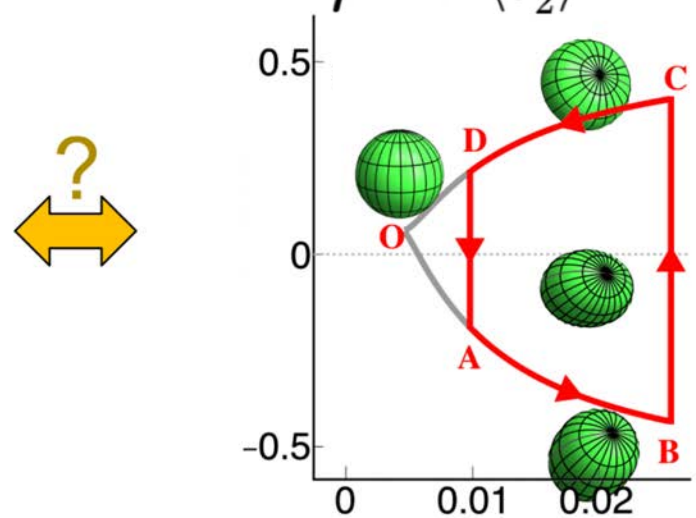
$$\langle\varepsilon_2^2\delta d_\perp/d_\perp\rangle \approx [0.0005-(0.07+1.36\cos(3\gamma))\beta_2^3]\times 10^{-2}$$

$$\rho = \frac{\langle\varepsilon_2^2\delta d_\perp\rangle}{\langle\varepsilon_2^2\rangle\sqrt{\langle(\delta d_\perp)^2\rangle}}$$

Map from (β_2,γ) plane to HI observables

How about $\langle v_2^2\delta[p_T]\rangle$ vs. $\langle v_2^2\rangle$

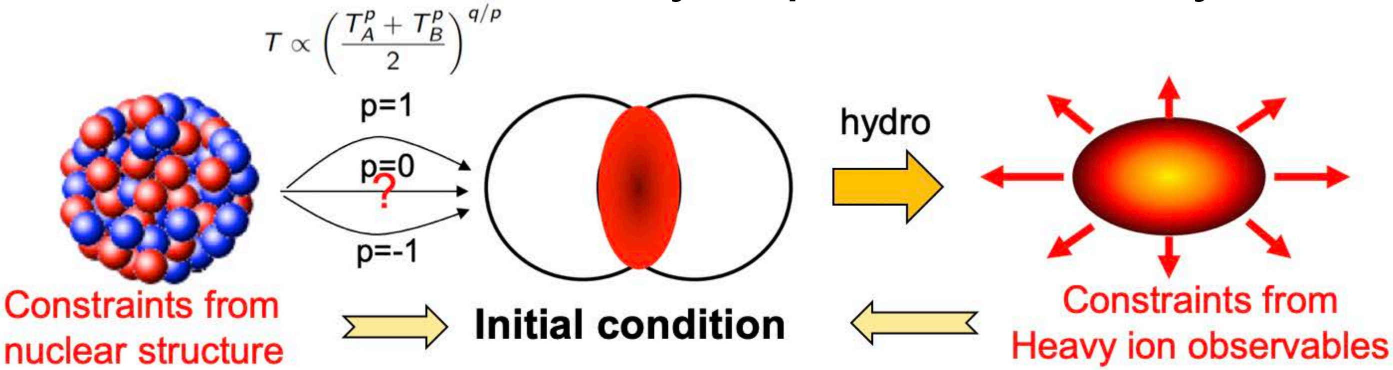




Collision system scan to map out this trajectory: calibrate coefficients with species with known β,γ, then predict for species of interest.

What future collisions can bring?

Future well-motivated system-scan to use heavy-ion collisions into a precision tool for both nuclear structure and initial condition, and ultimately improve the study of QGP.



More than 250 stable (long-lived nuclei), about 140 of them in isobar pairs or triplets

A	isobars	A	isobars	A	isobars
36	Ar, S	106	Pd, Cd	148	Nd, Sm
40	Ca, Ar	108	Pd, Cd	150	Nd, Sm
46	Ca, Ti	110	Pd, Cd	152	Sm, Gd
48	Ca, Ti	112	Cd, Sn	154	Sm, Gd
50	Ti, V, Cr	113	Cd, In	156	Gd, Dy
54	Cr, Fe	114	Cd, Sn	158	Gd, Dy
64	Ni, Zn	115	In, Sn	160	Gd, Dy
70	Zn, Ge	116	Cd, Sn	162	Dy, Er
74	Ge, Se	120	Sn, Te	164	Dy, Er
76	Ge, Se	122	Sn, Te	168	Er, Yb
78	Se, Kr	123	Sb, Te	170	Er, Yb
80	Se, Kr	124	Sn, Te, Xe	174	Yb, Hf
84	Kr, Sr, Mo	126	Te, Xe	176	Yb, Lu, Hf
86	Kr, Sr	128	Te, Xe	180	Hf, W
87	Rb, Sr	130	Te, Xe, Ba	184	W, Os
92	Zr, Nb, Mo	132	Xe, Ba	186	W, Os
94	Zr, Mo	134	Xe, Ba	187	Re, Os
96	Zr, Mo, Ru	136	Xe, Ba, Ce	190	Os, Pt
98	Mo, Ru	138	Ba, La, Ce	192	Os, Pt
100	Mo, Ru	142	Ce, Nd	198	Pt, Hg
102	Ru, Pd	144	Nd, Sm	204	Hg, Pb
104	Ru, Pd	146	Nd, Sm		

What future collisions can bring?

Future well-motivated system-scan to use heavy-ion collisions into a precision tool for both nuclear structure and initial condition, and ultimately improve the study of QGP.

Representative cases were identified in a dedicated EMMI taskforce among HI and structure experts.

https://indico.gsi.de/event/14430/contributions/64193/

The discussion lead to the identification of three science cases that may readily lead to breakthrough observations via relativistic collision experiments. They involve nuclides belonging, respectively, to the mass regions $A \sim 20$, $A \sim 40$, and $A \sim 150$. A~200

Stress test small system collectivity with extreme deformability

Imaging strong shape evolution of $^{144-154}$Sm isotopic chain

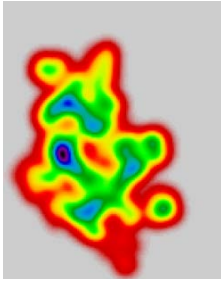
Exploiting the lever-arm provided by nuclear structure

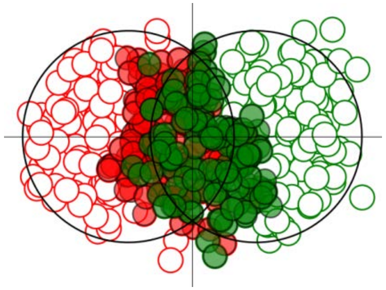
Flow observable = k ⊗ initial condition (structure)

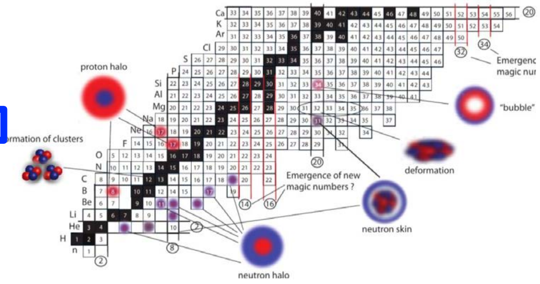
QGP response,
a smooth function of N+Z

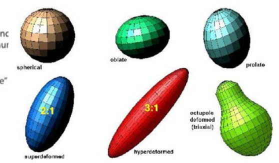
Structure of colliding nuclei,
non-monotonic function of N and Z

QGP





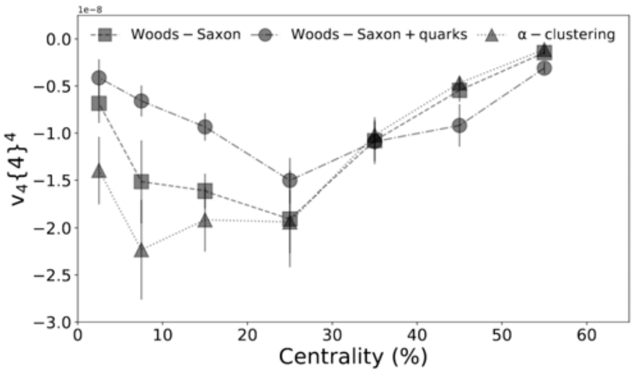


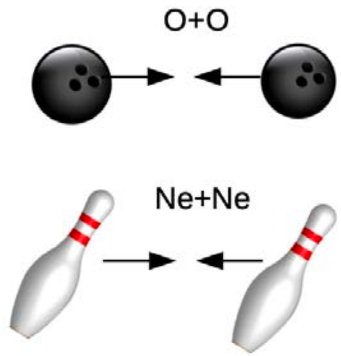


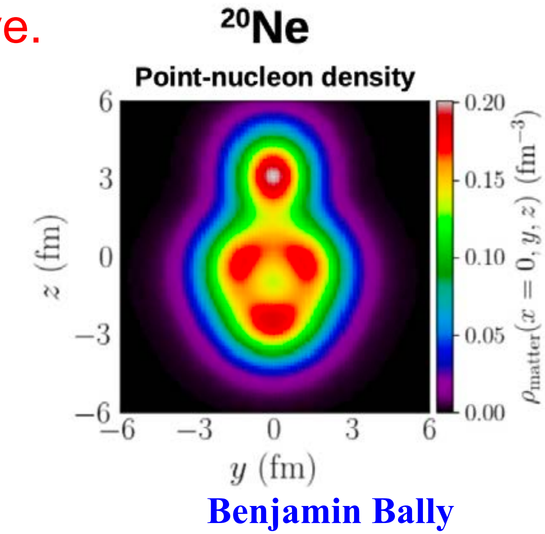
Stress-testing small system collectivity with 20Ne

Geometric origin of small system program is rather qualitative.

Very subtle effects if one only have 16O+16O collisions

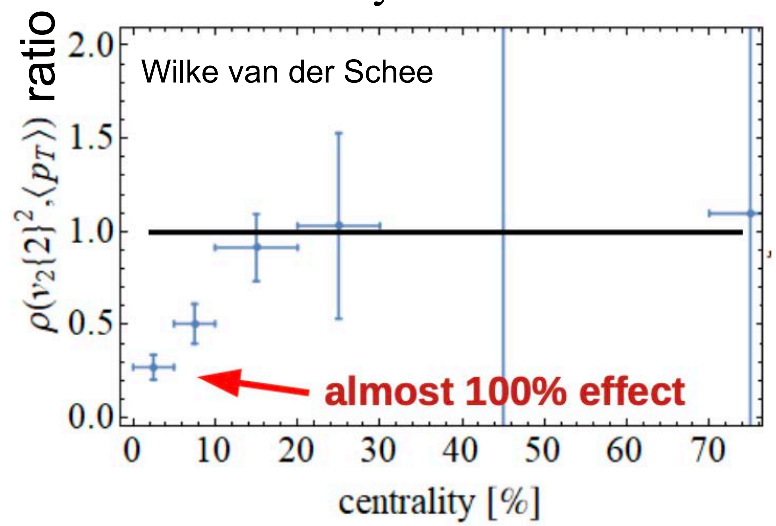






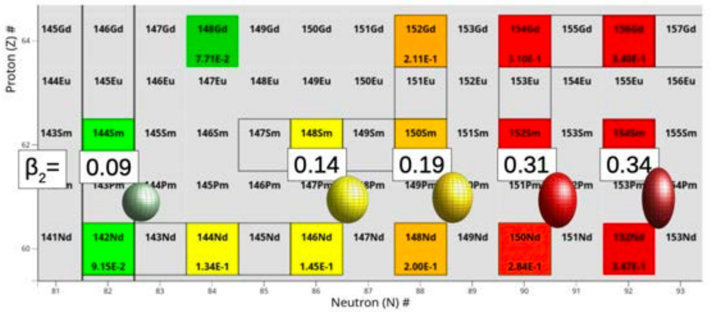
20Ne with 5 alpha has the most extreme ground state shape: $\beta_2 \sim 0.7, \beta_3 \sim 0.5$. Use O+O and Ne+Ne to observe "strong" purely-geometric effects at dN/dy~100.

Trajectum framework
Ratio 20Ne+20Ne /16O+16O



Imaging strong shape evolution of $^{144-154}$Sm isotopic chain

Transition from nearly-spherical to well-deformed nuclei when size increase by less than 7%. Using HI to access the multi-nucleon correlations leading to such shape evolution, as well as dynamical β_3 and β_4 shape fluctuations.



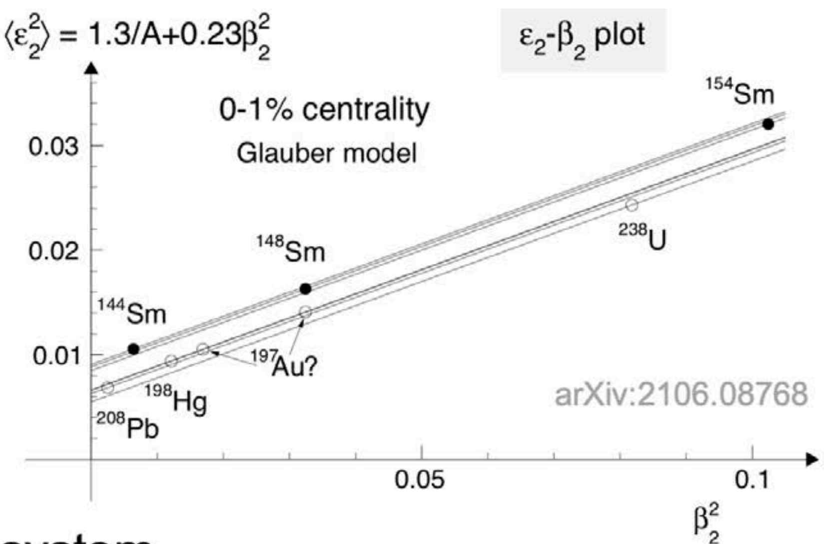
$$\left\langle \epsilon_2^2 \right\rangle = a' + b'\beta_2^2$$

$$\left\langle v_2^2 \right\rangle = a + b\beta_2^2$$

In central collisions

$$a' = \left\langle \varepsilon_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$

$$a = \left\langle v_2^2 \right\rangle_{|\beta_2=0} \propto 1/A$$



b', b are ~ independent of system

Systems with similar A fall on the same curve.

Fix a and b with two isobar systems with known β_2, then make predictions for the third one

Application in ^{197}Au+^{197}Au vs ^{238}U+^{238}U

arXiv:2105.01638

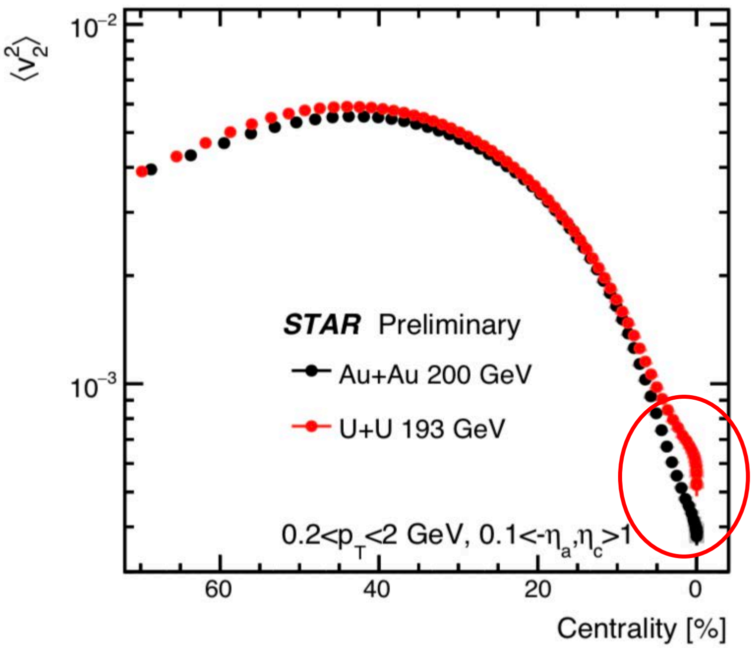
Ultra-central Collisions at $\sqrt{s_{NN}}$=193-200 GeV

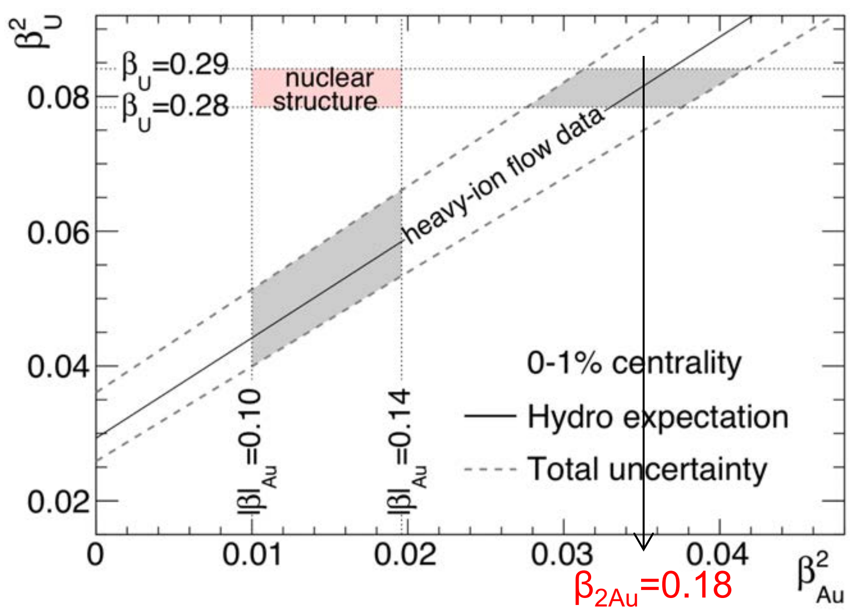
$$\begin{cases} v_{2,\mathrm{Au}}^2 = a_{\mathrm{Au}} + b\beta_{2,\mathrm{Au}}^2 \\ v_{2,\mathrm{U}}^2 = a_{\mathrm{U}} + b\beta_{2,\mathrm{U}}^2 \end{cases}$$

b~0.014

Need to correct for slightly different size: $a \propto 1/A\,,\quad r_a = \frac{a_{\mathrm{Au}}}{a_{\mathrm{U}}} = \frac{238}{197} = 1.21$

A linear relation for β_{2U} and β_{2Au} : $\beta_{\mathrm{U}}^2 = \frac{r_{v_2^2} r_a - 1}{b/a_{\mathrm{U}}} + r_{v_2^2}\beta_{\mathrm{Au}}^2 \quad r_{v_2^2} = \frac{v_{2,\mathrm{U}}^2}{v_{2,\mathrm{Au}}^2}$





Suggests $|\beta_2|_{Au}$~0.18+-0.02, larger than NS model of 0.13+-0.02

Summary

- Constrain QGP initial condition with nuclear structure input and heavy ion observables
  ➔ Improve the extraction of QGP properties in the Bayesian approaches
- Understanding how initial condition responds to nuclear structure, in turn enables probing novel nuclear structure properties and compliments low-energy experiments.
- Collisions of carefully-selected isobar species will help us to understand the many-body nucleon correlations of atomic nuclei from small to large system

A lot of work is needed to firm-up this science case

PROGRAM

JANUARY 23 - FEBRUARY 24, 2023

Intersection of nuclear structure and high-energy nuclear collisions (23-1a)

INSTITUTE for NUCLEAR THEORY

Organizers:
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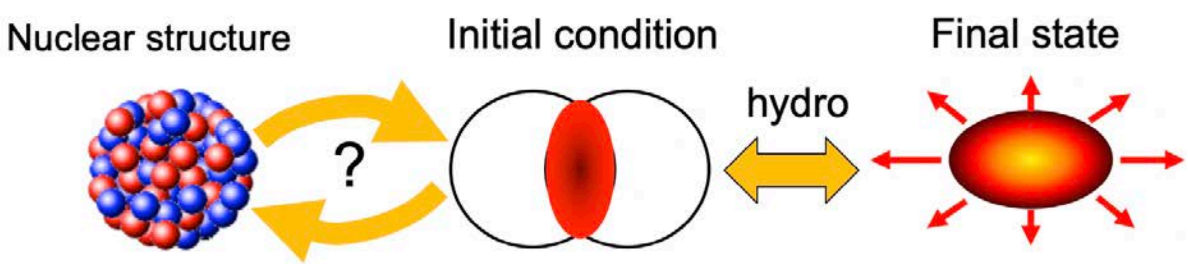
Jiangyong Jia (Stony Brook & BNL)
Dean Lee (Michigan State & FRIB)
Matt Luzum (São Paulo)
Jaki Noronha-Hostler (Urbana-Champaign)
Fuqiang Wang (Purdue)



arXiv:2102.08158

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